

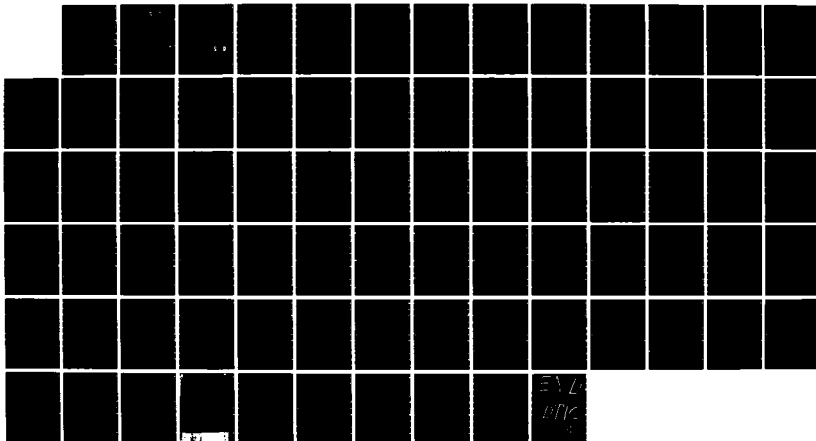
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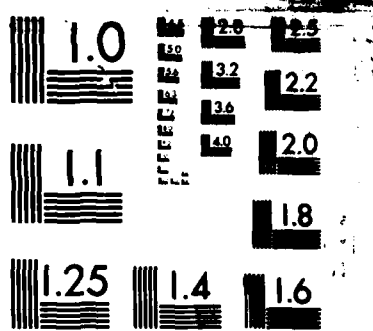
ANALYSIS OF THE REMAINING STRENGTH OF CONCRETE JACKETED 1/1
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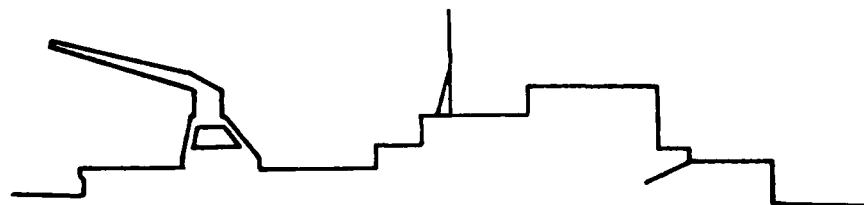
ANALYSIS OF THE REMAINING STRENGTH

OF

CONCRETE JACKETED STEEL H-PILES

FPO-1-82(12)

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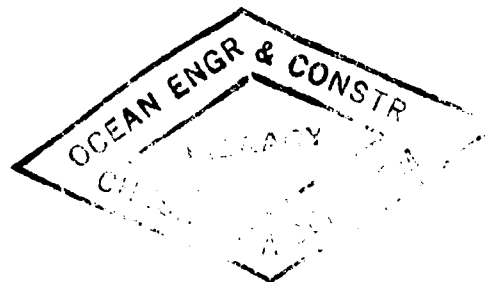
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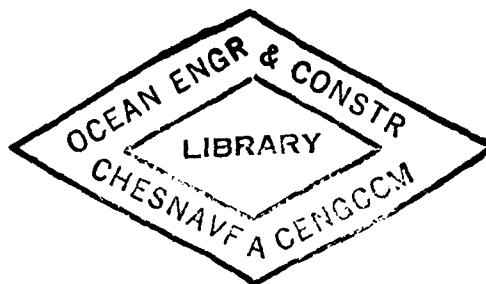
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A method for calculating the remaining strength of partially concrete jacketed H-piles has been developed. Using tables and graphs contained in this report the capacity of sample piles in Piers G and H at Charleston Naval Shipyard were determined.

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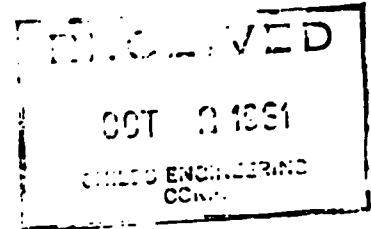
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IN SUPPORT OF
UNDERWATER FACILITY ASSESSMENTS AT VARIOUS LOCATIONS

CONTRACT N62477-80-C-0102
MODIFICATION P-00006

SCOPE OF WORK
FOR
TASK NO. 7.0



III. ADDITIONAL SPECIFIC REQUIREMENTS

Task 7 consists of engineering services necessary to provide an assessment, comparison and review of current engineering structural analysis techniques for steel H-piles. The main effort should be to achieve an optimum method of structural analysis incorporating a realistic factor of safety. The relationship of deterioration and loss of cross-sections over time and how this influences the bearing capacity and mode of H-pile failure should be part of this examination.

Review of the Charleston Naval Shipyard (CNSY) report of 1 Dec 1978 entitled "Evaluation of Capacity of Existing Piles - Pier G" will be made, analyzing the method of structural analysis and establishing a broader more firm basis of support for final recommendations to be made in the current report on the CNSY.

The structural analysis methodology or technique arrived at from this study should be incorporated into the current Underwater Facilities Inspection and Assessment at CNSY Report and may influence its findings. The overall Task 7 findings should be published in a concise report.

A. Completion Date:

Task 7 will be completed 90 days after notice to proceed.

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Executive Summary

A method for calculating the remaining strength of partially concrete jacketed H-piles has been developed. Using tables and graphs contained in this report the capacity of sample piles in Piers G and H at Charleston Naval Shipyard were determined.

> to p. 2

The strength of all the piles in Piers G and H are equal to or in excess of the 51.8 tons maximum required capacity.

Project Outline

In accordance with Contract No. N62477-80-C-0102, Modification P-00006, Task No. 7.0, the report which follows is submitted.

The objective of this investigation, has been to determine a reasonably accurate method for calculating the remaining strength of a steel H-pile which has a concrete jacket over part of its length and whose exposed surfaces have been diminished in thickness by the corrosive action of sea water. *Keywords: Strength analysis.*

Figure 1 shows the configuration of the pile with its concrete jacket.

Several conditions have been assumed based on generally accepted engineering practice. These assumptions may be worthy of further study and, as a matter of fact, a number of papers have been published concerning some of these assumptions. It has been assumed that the top of the pile is imbedded sufficiently deep in the concrete pile cap to guarantee a rigid connection between the pile and the pile cap. It is also assumed that the pile penetrates materials of sufficient density to assume fixity at 5' below the dredge line. It was also assumed that the concrete jacket around the pile was not integrally connected to the pile cap. During the investigation, it was found that, whether or not the jacket is connected to the pile cap has no significant effect on the overall load carrying capacity of the pile.

Three major considerations were considered important enough to study:

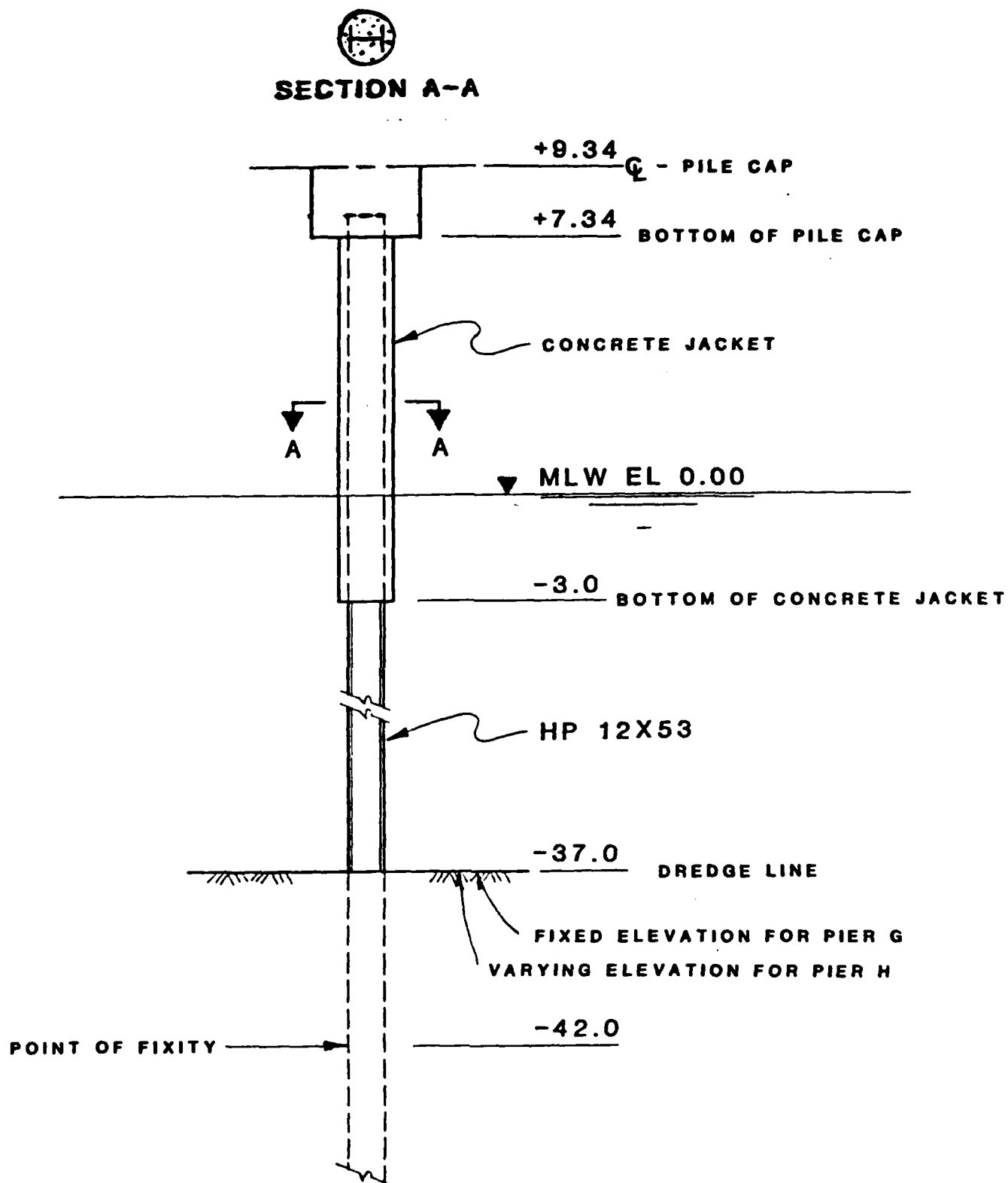
- I. The effect of the concrete jacket at the top of the pile on column stiffness.
- II Effect of deterioration of the pile in general on the overall load carrying capacity.
- III Effect of local deterioration on local buckling strength of the column.

Besides the three conditions enumerated above, the stiffness of the concrete pile cap spanning between piles was investigated during the computer portion of the study. It was determined that its stiffness normal to its axis was so high it had no appreciable affect on the overall stiffness of the pile.

Background

The object of this investigation is to determine if a simple, straight forward, accurate method for determining the remaining capacity of the subject pile has been or could be developed.

The first step was to conduct a literature search to determine if the problem at hand had ever been directly addressed. A search through personal libraries, local engineering school libraries, the National Technical Information Service and Compendix files was made. A great deal of information possibly addressing itself to the problem was collected. Unfortunately, no one had directly addressed the problem in any of the literature found. Results and conclusions of this report, therefore, are based on parts of the literature found, the use of a finite element analysis computer program and basic structural analysis.



NOTE: THE DIMENSIONS AND SIZES SHOWN
ARE TAKEN FROM DRAWINGS OF THE PILES
IN PIERS G AND H AT THE CHARLESTON
NAVAL SHIPYARD IN CHARLESTON, S.C.

TYPICAL PILE

FIGURE 1

Methodology

We investigated by finite element (computer) analysis and classical mathematical analysis the critical buckling strength of a steel H-pile, with and without a concrete jacket. The two methods were used to enable us to test the computer software against known values.

The first check was of a simple H-pile fixed at both ends and having a constant moment of inertia. The value of critical buckling obtained from the computer model was within 0.3% of the mathematical value.

Next, a concrete jacket was assumed to be on the upper portion of the pile having a combined moment of inertia 1380 percent greater than that of the bare pile. The critical buckling was computed by finite analysis and according to formulae given by Roark.¹ The results compared within 2%. These two tests satisfied us that the finite element software was sufficiently accurate for our further study. (See Appendix A)

Since the moments of inertia of the piles and jackets vary, overall pile capacities were calculated two ways for Pier H. The first method used the results of the finite element analysis. From the finite element analysis it was determined that if the concrete jacket was sound, it reduced the length of the pile to that distance between the bottom of the jacket and the assumed point of fixity 5' below the mudline.

The reduced moment of inertia of the portion of the H-pile below the jacket was determined by averaging the reduced moments of inertia along the length of the pile wherever measurements had been taken. In the URS/Madigan-Praeger Inc. Report regarding Pier G, measurements were shown to have been taken just above the mudline, just below the bottom of the concrete jacket and halfway between these two. In the Childs Engineering Report regarding Pier H at the same facility, measurements were taken along the full exposed length of the pile in some instances and three locations below the bottom of the jacket in others. In order to eliminate large variations the least dimension and the greatest dimension on each pile were eliminated from the averaging of the moments of inertia. Local buckling calculations were made using the least dimensions measured on each pile.

In the first method of analysis the stiffness factor "K" used in the Euler Formula was used as a constant value of 3.9. This value was arrived at from the computer analysis (See Appendix A). This value is useable when the concrete jacket results in a moment of inertia at least 13.8 times greater than the moment of inertia of the unprotected portion of the pile when the jacket is approximately 25 percent of the total length of the pile.

Since the piles considered had effective lengths and radii of gyration such that they act as columns in the inelastic buckling range (le/r less than 120), the tangent modulus (E_t) was used in computing their overall buckling strength. In accordance with a method developed by Avi Mordkowitz published in "Machine Design" on

October 3, 1974 entitled "Safe loads for inelastic buckling", the critical stress for each pile was determined using a single calculation rather than the usual trial-and-error process. (See Appendix B) Figure 2 shows a curve representing the value of tangent modulus versus stress for A-7 steel. The curve is used in accordance with the method worked out by Mordkowitz.

Local buckling was investigated using the American Institute of Steel Construction Specifications for allowable width versus thickness of flanges and webs of H-pile sections. Wherever the width over thickness ratios exceeded the values given by A.I.S.C., critical local buckling stresses were calculated in accordance with Article 65 of Formulas for Stress and Strain Fourth Edition, by Raymond J. Roark.¹

Using factors of safety based on the end conditions and the buckling curves as specified by A.I.S.C., the allowable overall column buckling strengths were calculated. Local buckling strength where applicable for each column were also calculated. The minimum value for either column or local buckling was used as the limiting value for that pile.

The second method of overall buckling analysis used an extrapolation of the values for $E_2 I_2 / E_1 I_1$ and a/l given in Table 34 No. 1e of Roark⁴ as shown on Figure 3. Using the comparative moments of inertia and the ratio of jacketed length to total length, a factor "K" can be determined. This is the same factor used in the Euler formula except that its value may exceed 4.0 because it accounts for not only theoretical shortening of the actual pile length but also an increase in the effective moment of inertia.

Once the "K" factor is determined, solution of the Euler equation using Young's modulus (E) or the Tangent modulus (E_t) was the same as in the first method described above.

Local buckling computation and comparison of the two types of buckling to determine which controls was carried out as in method one.

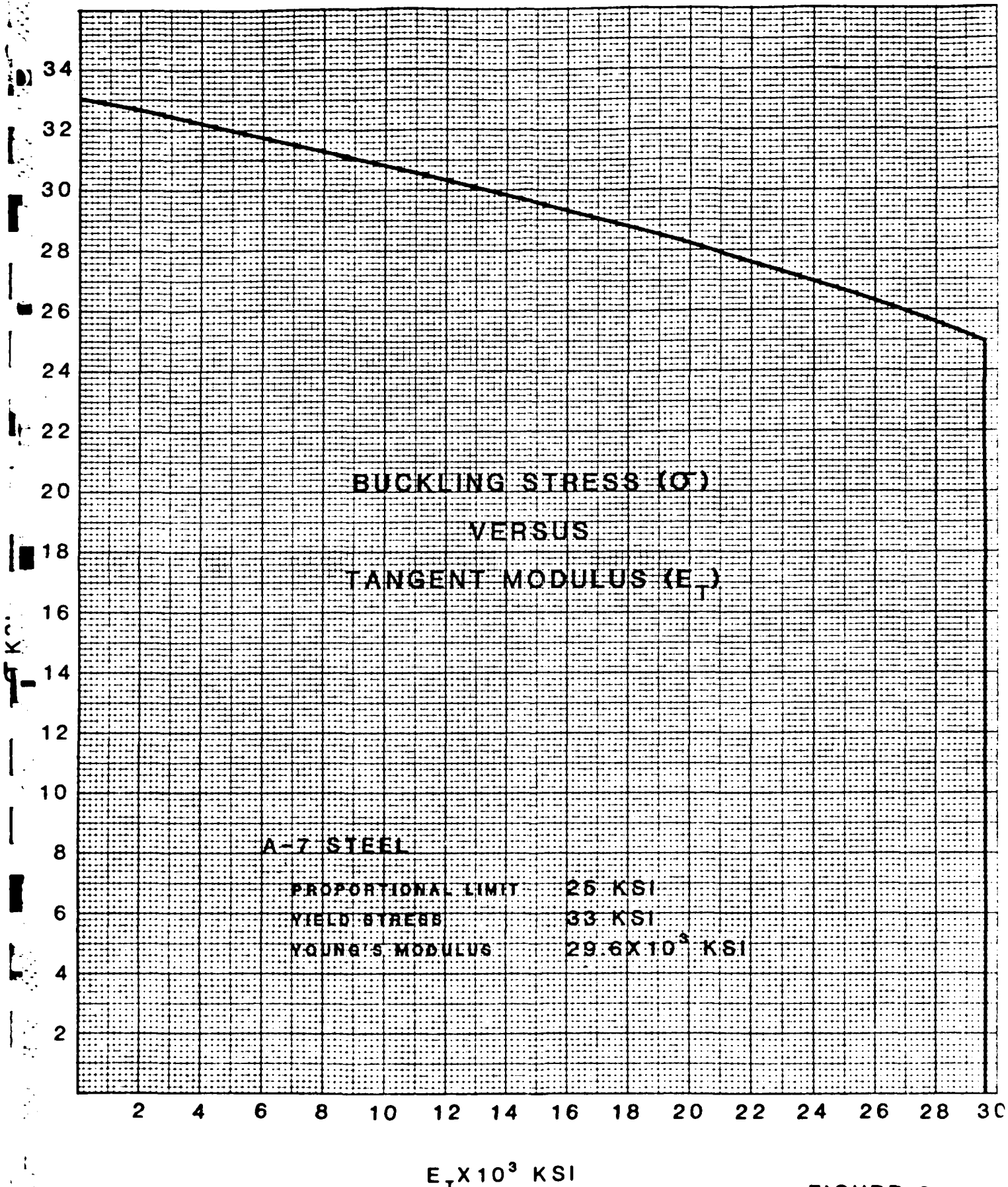


FIGURE 2

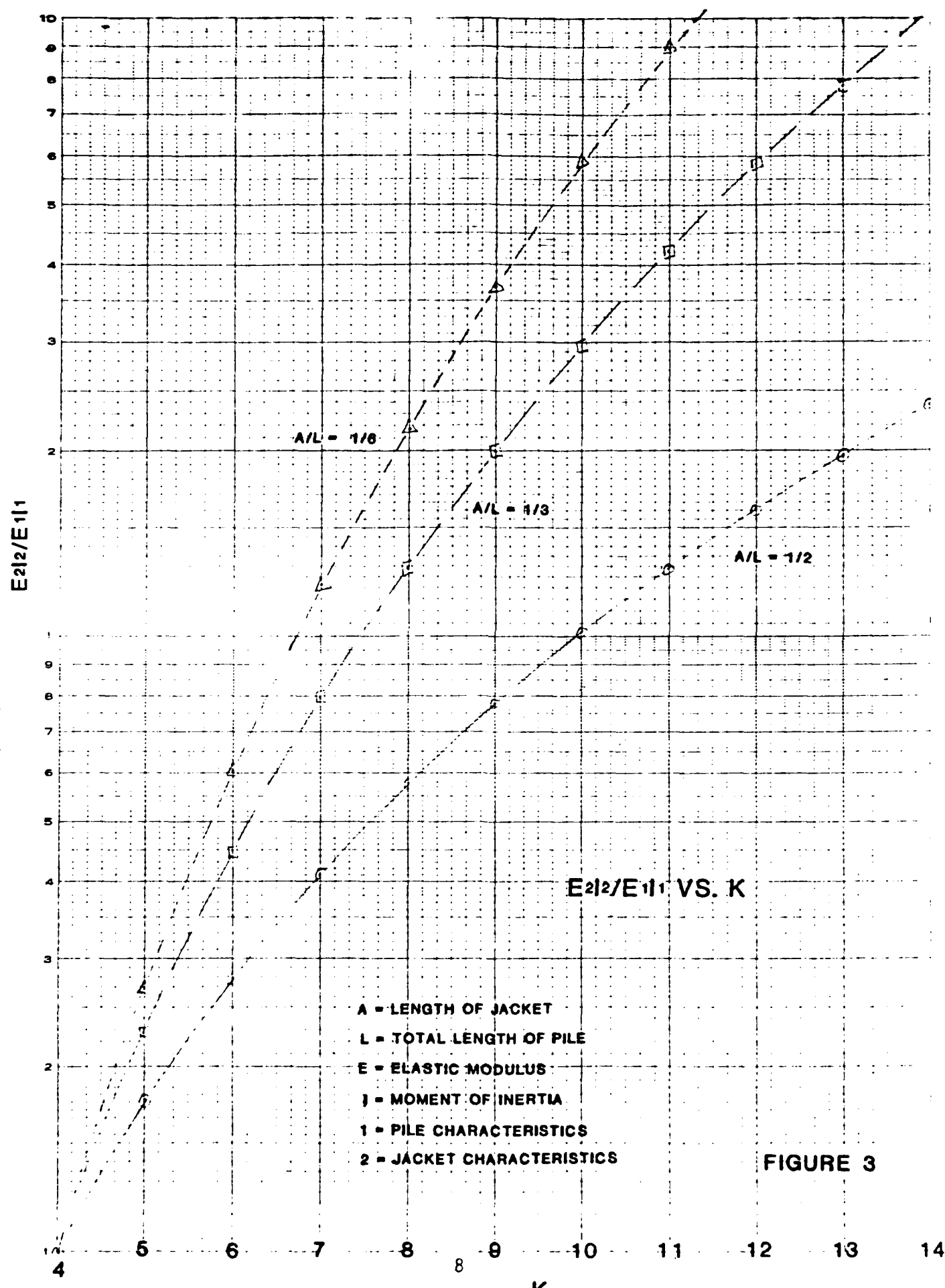


FIGURE 3

Factors of Safety

Effective column lengths (KL) used in the Euler formula for calculating critical elastic buckling loads in "long" columns are derived from considerations of the behavior of a column having various end restraints. A column pinned at both ends is the basic column and has an effective length of L. A column with one end free and the other end fixed, because of symmetry, behaves in exactly the same way as a pinned column twice as long; therefore, the effective column length would be $2L$. A column fixed at both ends has an effective length of $\frac{1}{2}L$. Other configurations using other end restraints yield different effective lengths.

In practice the effective column lengths used are somewhat longer than the above theoretical values. As in all engineering considerations, a factor of safety is incorporated in the determination of effective length multipliers. The column formulae presume that the column is initially straight, that the column material is homogeneous and that the loading is concentric.

The above considerations together with the realization that residual stresses in steel due to cooling after welding or hot rolling or some other fabrication operation may lower the buckling load levels require the use of factors of safety. "The critical stress to be expected under any actual set of circumstances is nearly always less than indicated by the corresponding theoretical formula, and can only be determined with certainty by test."¹

Once the critical stress is calculated according to the appropriate formula a factor of safety is applied to take into account the anomalies mentioned above. It should be recognized that the modification of the factor "K" from theoretical to assumed values used to adjust for the effective length of the column $l_e = Kl$ is, in actuality, the application of another factor of safety.

The A.I.S.C. in the latest edition of its Manual of Steel Construction (1980) limits the ratio of the width of a flange and its thickness of a steel H-pile to a value of $\frac{95}{\sqrt{F_y}}$. When the ratio exceeds this value reduction factor must be applied to the maximum allowable stress to get the useable stress. In this study, this reduction factor (Q_s) is considered a factor of safety. A representative value for the piles being considered is 0.78. This value results in a factor of safety of 1.28.

In addition, the A.I.S.C. recommends a varying factor of safety for determining allowable buckling stress from calculated critical buckling stress curves. This factor of safety varies from 1.92 to 1.67 depending where on the composite Engesser/Euler slenderness curve the column is located. (See Table I) The lower value of 1.67 was used since the columns analyzed are in the inelastic or Engesser portion of the slenderness curve.

Combining the two factors of safety 1.28 and 1.67 gives an overall factor of safety of 2.14. This value was used in determining allowable overall buckling stress based on critical overall buckling stress.

Local column buckling formulae for determining critical local buckling stresses as developed by the A.S.C.E. Column Research Council incorporate a factor of safety of 1.67. Local buckling is less subject to material non-homogeneity and eccentric loading because of the restrictive areas over which this type of buckling acts. This factor of safety which is lower than the overall column buckling factor of safety is deemed adequate for the reasons cited above. In field examinations local buckling due only to loss of section through corrosion has not been found to constitute the failure mode for a column.

Table 1

A.I.S.C. Column Allowable Stresses

C_c = Column slenderness ratio dividing elastic and inelastic buckling.

$$C_c = \pi \sqrt{\frac{2E}{F_y}}$$

E = Young's modulus
F_y = Yield stress

For A-7 steel

E = 29,600,000 psi
F_y = 33,000 psi

$$C_c = 133.1$$

For A-36

E = 30,000,000 psi
F_y = 36,000 psi

$$C_c = 128.25$$

Factors of Safety

$\frac{kL}{r} > C_c$ Elastic buckling range

$$F_s = 23/12 = 1.92$$

$0 < \frac{kL}{r} < C_c$ Inelastic buckling range

$$\frac{kL}{r} = 0, F_s = 5/3 = 1.67$$

$$\frac{kL}{r} = C_c, F_s = 23/12 = 1.92$$

$\frac{kL}{r} = 0$ Short columns

$$F_s = 5/3 = 1.67$$

See reference 2

$$95.0/\sqrt{F_y} < \frac{b}{t} < 176/\sqrt{F_y}$$

$$Q_s = 1.415 - .00437 \left(\frac{b}{t}\right) \sqrt{F_y} = 0.78 \text{ (representative value)}$$

$$\frac{1}{Q_s} = 1.28$$

Steel Pile Corrosion

The prediction of the steel corrosion deterioration requires at least two examinations with a time interval between two to five years in order to predict future corrosion. The initial coatings of the piles would have protected the pile against rapid corrosion until it deteriorated. Assuming corrosion at the present rate started the day the piles were driven leads to the assumption of a lesser than actual rate for loss of metal. Base line piles which are located and carefully evaluated during each periodic inspection will allow the prediction of corrosion rate.

Condition I

The addition of a concrete jacket around a steel bearing pile may have been done for two reasons. First, usually when added during initial construction it is intended to protect the pile against corrosion. Since most salt water corrosion occurs from a few feet below low water to the top of the splash zone, the jacket starts at the underside of the pile cap and extends a few feet below low water. Another technique calls for the concrete jacket to extend from the pile cap to the sea floor.

Second, when the jacket is added after the pile has been in place for sometime, it is being used to strengthen the corroded pile and to protect the pile from further corrosion. The jacketing usually extends from a few feet below low water up through the splash zone. It is not attached to the pile cap and may or may not contain reinforcing.

This discussion addresses itself to the pile jacket which occurs only at the top of the pile and which is an integral part of the pile cap. Jacketing of piles for their full length adds to their stiffness but, since it is for the full length of the pile, there is no discontinuity to consider.

The two foot diameter concrete jacket added to the top of the steel H-piles under investigation increases the minor axis moment of inertia better than 12 times. The finite element analysis indicates that this is tantamount to shortening the pile for the length of the jacket. From Figure 1e, table 34 of Formulas for Stress and Strains, Fifth Edition by Roark and Young, (See Appendix B) it can be seen that as the moment of inertia of the jacket increases with regard to the moment of inertia of the pile, the critical buckling load increases. As the length of the sound pile jacket increases, the critical buckling strength of the pile also increases.

Our concern that the jacket may not have been fully bonded to the pile cap was addressed in the computer model. Leaving a theoretical one inch gap between the bottom of the pile cap and the jacket made no significant difference in the overall capacity of the pile. This was not surprising, inasmuch as the steel pile was embedded sufficiently into the concrete cap to develop full practical fixity. The concrete pile jacket even if it were bonded to the pile cap would still not improve on the fixity. Vertical cracks in the pile jacket, since they do not effect the moment of inertia of the cross section of pile jacket and pile, have no theoretical effect on the moment of inertia. Horizontal cracks (cracks normal to the longitudinal center line of the pile) would have effect only if they were numerous so that the concrete jacket could not act as a continuous member. In the actual inspection, few horizontal cracks were found. (See Note 4. in the Addendum for further discussion of relationship of concrete jacket and steel H-pile)

The mathematical method for calculating the critical load of a steel H-pile which is partially protected by a round concrete jacket is as follows:

1. Determine the original profile of the steel H-pile and its original cross-sectional area, moment of inertia about the weak axis and resulting minimum radius of gyration.
2. Measure the length of the concrete jacket and the overall length of the pile from the bottom of the pile cap to the mudline.
3. Measure the diameter of the concrete jacket. Compute the combined moment of inertia of the jacket and H-pile about the weak axis of the H-pile.
4. Measure the loss of cross-sectional area of each flange and the web of the H-pile at the mudline, just below the concrete cap, at mid-height and at any other location of severe corrosion.
5. Compute average area and remaining moment of inertia about minor axis over exposed length of pile and minimum area and moment of inertia about minor axis.
6. Using $\sigma_{cr} = \frac{K \pi^2 E}{(1/r)^2}$ find critical buckling load using either Young's modulus (E) or Tangent modulus (E_t) as suits the column conditions. Column factor K is to be determined using either the results of a finite element analysis (as done in this report) or an extension of Table le from Formulas for Stress and Strain 5th Edition by Roark and Young as shown in Figure 3.

If E_t is used, the method for determining the values of E_t and σ as shown in "Safe Loads for inelastic buckling" by Mordkowitz should be used. The curve of σ versus E_t for A-7 steel is included as a part of this report in Figure 2

The length of pile used in the formula depends upon the comparative moments of inertia of the jacketed portion of the pile and the bare portion. If the jacketed portion is very much stiffer, it is considered a part of the pier deck structure and the length of pile considered is the unjacketed length. (The condition found in this study was $I_2 = 24I_1$ when the piles were new. $K=3.9$ takes into consideration the slight flexibility of the jacket when compared to a completely rigid structure.)

If the jacket were only to increase the moment of inertia in that area by a factor less than 24 the relationships shown

in Figure 3 would be used for K.

7. Using appropriate factors of safety dependent upon flange width to thickness ratios and overall buckling condition (in accordance with A.I.S.C.) calculate allowable column buckling load.
8. Calculate if local buckling need be considered. If it does, calculate allowable loads considering local buckling. Using the cross-section of the pile where the greatest corrosion has occurred, calculate the ratios of width versus thickness (b/t) for the flanges and web. If these values exceed those allowed by the A.I.S.C., calculate the allowable buckling stress using the A.S.C.E. Column Research Council formulae, viz, $\sigma_{cr} = \frac{.416E}{1-\nu^2} \left(\frac{t}{b}\right)^2$ for flanges and $\sigma_{cr} = \frac{3.29E}{1-\nu^2} \left(\frac{t}{b}\right)^2$ for webs.
9. The lesser of the overall allowable buckling load or allowable local buckling loads determine the allowable column load.

Condition II

The overall critical buckling load of the pile was determined by using the length of the pile between the bottom of the concrete jacket and a point five feet below the harbor bottom. The critical buckling loads of a column having this length were determined using either the Euler Formula or Engesser Formula. Essentially these two formulae are the same except that the Euler Formula uses the Young's modulus (E) of the material while the Engesser Formula uses the Tangent modulus (E_t) of the material. (Many texts addressing the strength of columns discuss the difference between Young's modulus and Tangent modulus). All of the piles investigated in Pier H were short enough to be considered as short columns where the critical load was inelastic buckling load utilizing the Engesser Curve for calculating these loads. The test for the computer model assumed that the columns were acting as long columns which would fail due to elastic buckling. Their capacity was calculated using the Euler Curve. These values are not actually correct in an absolute sense, although for comparison purposes they are sufficient.

If the corrosion of a steel H-pile occurs at its midlength, the effect of the loss of cross section on the overall column buckling strength decreases as the affected length decreases. If the corroded area of a steel H-pile encompasses its full length, the loss of strength usually varies in direct proportion to its loss of area. This relationship is easily understood if one considers that when corrosion diminishes the thickness of the flanges and web, the loss of area and loss of moment of inertia are in direct proportion. Critical load on a column is directly proportional to that pile's moment of inertia. Bracket et al in their study¹¹ found that as the affected length of pile whose area was reduced was shortened, the effect on the overall column capacity diminished. Example: If a pile had lost 50% of its cross sectional area due to corrosion its overall column load carrying capacity would be reduced 50% if the corrosion occurred over the full length of the pile. If the same loss in area had occurred over only the center 20% of the pile, the loss in overall column carrying capacity was only 30%.

In the following pages several characteristics of the piles in Pier H are calculated using dimensions as found in the Childs Engineering Corporation Report regarding Charleston Naval Shipyard¹³

Web_r = thickness of corroded H-pile web

$Avg FL_r$ = average thickness of the two flanges at any elevation where they have been reduced by corrosion

Web_{min} = minimum thickness of corroded web

FL_{min} = minimum thickness of corroded flange

A_r = remaining cross sectional area of the H-pile using Web_r and $Avg FL_r$ dimensions

I_r = remaining moment of inertia about the weak axis of the H-pile using Web_r and Avg FL_r dimensions.

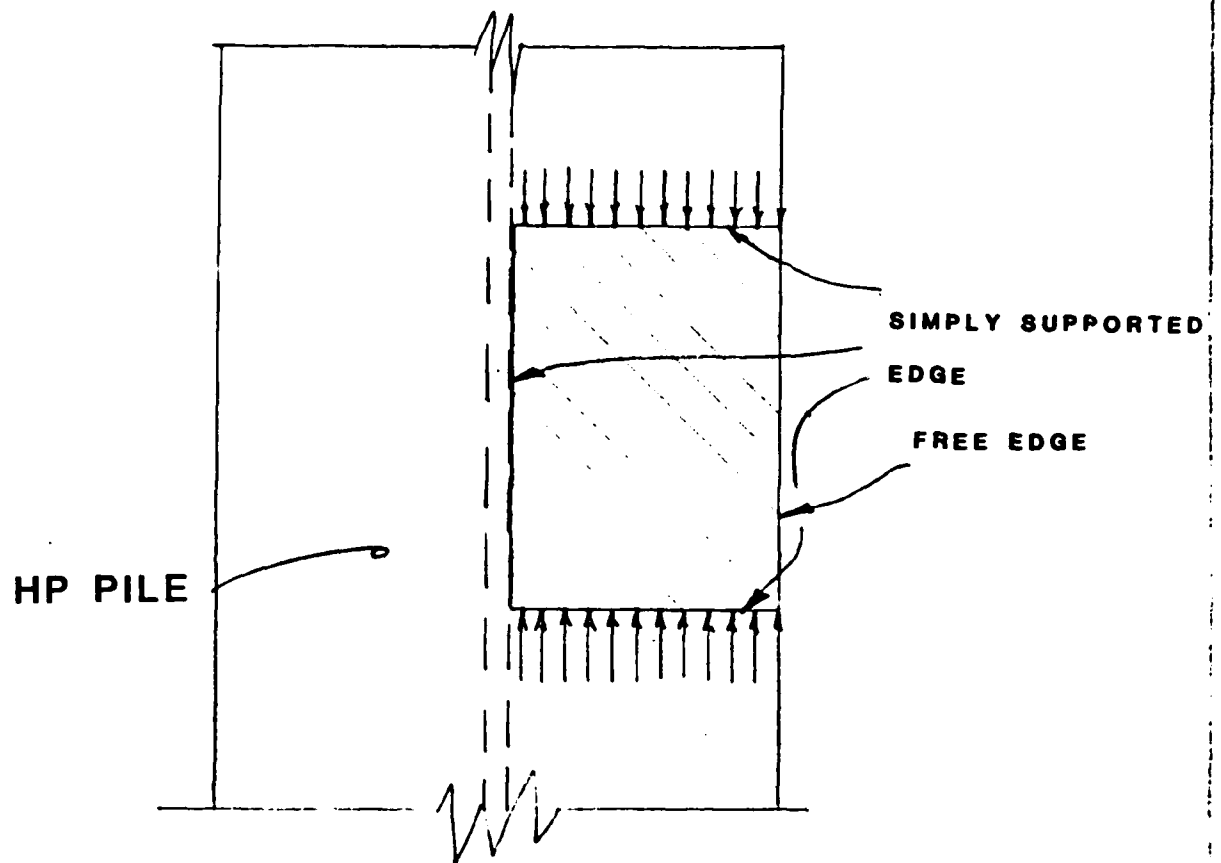
r = remaining radius of gyration about the weak axis of the H-pile using I_r and A_r .

$\frac{b}{t}$ = ratio of flange half width divided by FL_{min} .

$\frac{b_1}{t_1}$ = ratio of web width divided by Web_{min} .

Pile (xx) Identifier of pile.

l_r = actual length of pile from the bottom of the concrete jacket to the assumed point of fixity five feet below the actual mid line.



**EDGE RESTRAINTS
FOR
LOCAL BUCKLING**

FIGURE 4

OVERALL COLUMN BUCKLING

The calculations shown on the following pages were made to determine the overall critical buckling loads for the piles analyzed in Pier H. The term $\left(\frac{\pi}{L_e/r}\right)^2$ is described in Appendix C and is used in calculating the critical buckling stress σ_{cr} .

Pier H Method One

CRITICAL COLUMN STRESSES

II Overall Column Buckling

$$P_{cr} = K \frac{\pi^2 E_t I}{l^2}$$

The columns investigated are in the inelastic buckling range ($l_e/r < C_c$)
Tangent modulus, E_t , must be used to calculate P_{cr} & σ_{cr}

$$\sigma_{cr} = K \frac{\pi^2 E_t}{(l_e/r)^2} = \frac{\pi^2 E_t}{(l_e/r)^2}$$

$$\frac{\pi^2}{(l_e/r)^2} = \text{slope of } \sigma_{cr}$$

A = area of section

P_{cr} = Column Buckling Load

K = factor based on column end conditions = 3.9

E = tangent modulus

I = minimum moment of inertia

l = length of column

k = factor used to determine effective column length

$$k = \frac{1}{K}$$

l_e = effective length of column
 $l_e = k l$

σ_{cr} = Critical column buckling stress
 C_c = A.I.S.C. slenderness ratio for A-7 steel = 131.7

Pile	r	l	$\frac{\pi^2}{(l_e/r)^2}$	A	σ_{cr}	P_{cr}
9K	2.76 in	22.0 ft	.00421	10.76 in ²	31.4 ksi	338 k
19D	2.86	14.2	.01084	9.36	32.4	303
45K	2.90	26.0	.00333	13.1	30.9	405
1K	2.84	31.0	.00225	12.8	30.0	378
9A	2.86	25.2	.00345	11.8	31.1	364
59A	2.93	15.5	.00957	12.4	32.2	399
49B	2.82	20.2	.00522	12.0	31.7	380
34 ¹	2.91	19.2	.00615	11.2	31.9	357
24 A	2.83	20.8	.00496	11.2	31.6	354
19B	2.85	20.8	.00502	10.1	31.6	319

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75E-80 - Charles H. Hovell
SHEET NO. 1 of 2
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CHECKED BY: DLP DATE: 4/26/82
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Method Two

PIER HOTEL - COLUMN ANALYSIS

[Refer to Childs Engineering Report entitled "Analysis of the Remaining Strength of Concrete Jacketed Steel H-Piles", Feb. 1982]

PILE LOAD ANALYSIS

Note: Refer to Pier DELTA's Column Analysis for Definition of Terms used below.

Assumption: Steel is A-7 ($F_y = 33$ KSI)

Note: The columns investigated are in the inelastic buckling range ($l_e/r < C_c = 133$), so the tangent modulus, E_t , must be used to calculate column P_{cr} & σ_{cr} .

$$\sigma_{cr} = \frac{K \pi^2 E_t}{(l/r)^2} = \frac{\pi^2 E_t}{(l_e/r)^2} \quad P_{cr} = (\sigma_{cr})(A_r)$$

Jacket Stiffness:

a) HP 12 x 53: $\frac{E_2}{E_1} I_2 = I_{\text{effective concrete}} + I_{\text{HP 12x53}} = \left(\frac{\pi (24")^4}{64} \right) (.1) + 127 = 1756 \text{ in}^4$
 $\therefore \frac{E_2 I_2}{E_1 I_1} = \frac{1756 \text{ in}^4}{I_r}$

b) HP 12 x 74: $\frac{E_2}{E_1} I_2 = I_{\text{effective concrete}} + I_{\text{HP 12x74}} = \left(\frac{\pi (24")^4}{64} \right) (.1) + 185 = 1814 \text{ in}^4 \therefore \frac{E_2 I_2}{E_1 I_1} = \frac{1814}{I_r}$

PILE	9-K	19-D	45-K	1-K	9-A	59-A	49-B	34-1	24-A	19-B	
HP	HP 12x53	HP 12x53	HP 12x74	HP 12x53	HP 12x53	HP 12x53	HP 12x53	HP 12x53	HP 12x53	HP 12x53	
d_o (in)	.360	.275	.392	.378	.349	.320	.370	.287	.340	.302	
r (in)	.285	.265	.362	.361	.332	.370	.332	.335	.310	.285	
r (in ²)	10.76	9.36	13.10	12.78	11.77	12.37	12.00	11.16	11.15	10.13	
I_r (in ⁴)	821	76.3	110	104	95.6	110.7	95.6	96.5	89.3	82.1	
λ (in)	2.76	2.86	2.90	2.85	2.85	2.94	2.82	2.94	2.83	2.85	
L (in)	411	327	471	507	459	387	387	375	399	387	
λ	.36	.48	.34	.27	.31	.36	.37	.38	.35	.36	
$\frac{L}{\lambda}$	21.4	23.0	16.0	16.9	18.4	16.4	18.4	18.2	19.7	21.4	
$\frac{L}{\lambda}$	9.2	13.8	8.5	8.0	8.8	8.6	8.8	8.8	9.0	9.2	
$\frac{L}{\lambda}$.330	.269	.343	.354	.337	.341	.337	.337	.333	.330	
$\frac{L}{\lambda}$.00409	.0104	.00318	.00249	.00335	.00440	.00461	.00534	.00448	.00492	
σ_{cr} (ksi)	31.35	32.4	30.9	30.3	31.0	31.7	31.55	31.75	31.5	31.7	
P_{cr} (k)	337	303	405	387	365	392	379	354	351	321	

LOCAL BUCKLING

Condition III

The Critical load, P_{cr} , is the smallest load which will hold a column in an elastically buckled or slightly deflected form.

Some columns having unsupported flanges or legs which have a very small thickness to width (t/b) ratio may buckle locally and possibly this local buckling will reduce the overall buckling strength of the column.

A plate simply supported at its loaded edges, simply supported on one unloaded edge and unsupported at its other unloaded edge (this is the condition of any outstanding flange or a channel of H-beam or I-beam) will buckle at some load P_{cr} . However, as the load increases the stress close to the simply supported edge will increase. The stress at the mid section where the bulges form remains about constant even as the bulges continue to increase. Thus, the total load causing buckling failure will depend upon how much the bulges are allowed to deflect. Usually, the load is restricted to that which causes a stress in the neighborhood of the simply supported edge equal to the field strength of the column material. (See Figure 4.)

The A.I.S.C. gives relationships for the width to thickness ratios for flanges and webs of H-sections. For A-7 steel the width to thickness ratio for flanges should not exceed $\sqrt{\frac{3000}{F_y}}$ or 16.5 σ_y is the yield strength of material which is 33,000 psi. The width to thickness ratio of the web should not exceed $\sqrt{\frac{8000}{F_y}}$ or 44.0. As long as these two values are not exceeded local buckling will not control on a column. If these values are exceeded the stress computed as the load on the column divided by the net area should not exceed critical stresses calculated as follows:

$$\sigma_{cr} = \frac{.416E}{1 - \nu^2} \left(\frac{t}{b}\right)^2 \quad \text{For flanges}$$

$$\sigma_{cr} = \frac{3.29E}{1 - \nu^2} \left(\frac{t}{b}\right)^2 \quad \text{For webs}$$

Critical local buckling loads can than be calculated using these stresses and the reduced local areas.

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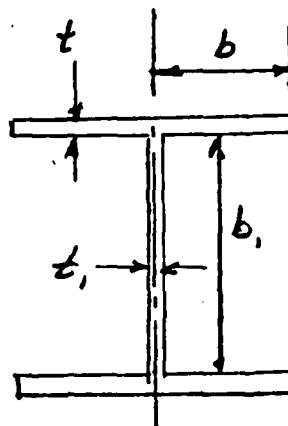
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Pier

Local Buckling - AISC

$$\frac{b}{t} > \frac{3000}{\sqrt{33000}} = 16.5$$

$$\frac{b_1}{t_1} > \frac{8000}{\sqrt{33000}} = 44$$



A.I.S.C. says if width over thickness ratios do not exceed the above values, local buckling will not occur. If these values are exceeded, higher stresses in accordance with given formulae are calculated. If local stresses do not exceed these calculated stresses, local buckling will not occur.

Pile	Flanges			Webs		
	b	t	b/t	b1	t1	b1/t1
9K	6.02"	.245	25 *	10.908"	.345	32
19D	↓	.240	25 *		.215	51 *
45K	6.108"	.300	20 *		.320	34
1K	6.023"	.330	18 *		.335	33
9A		.220	27 *		.295	37
59A		.365	16		.280	39.0
49B		.325	18 *		.320	34
34 #1		.305	20 *		.225	48 *
24A		.245	25 *		.295	37.0
19B	↓	.240	25 *	↓	.235	46 *

* Indicates that b/t exceeds allowable values

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Pier H

III Local Buckling Loads - Cont

Where b/t and b_1/t_1 allowable values are exceeded, calculate the allowable stress by use of A.S.C.E. Column Research Council formulae and allowable load by multiplying remaining area by this stress.

Pile	Flanges		Webs		EA	σ_{cr}	P_{cr}
	t	b	t_1	b_1			
9K	.215 in	6.023	.345 in	10.908	9.67 in ²	21.25 ksi	213 ^k
19D	.240		.215		8.13	21.09	171
15K	.300	6.108	.360		10.82	32.04	347
1K	.330	6.023	.335		11.60	11.98	462
9A	.220		.295		8.52	26.00	151
59A	.365	6 in	.280	10.908	11.85	49.38	578
49B	.325	6 in	.320	10.908	11.32	38.81	438
34 ^{#1}	.305		.225		9.80	9.44	334
21A	.245		.295		9.12	22.13	200
19B	.240		.235		8.35	21.25	176

Buckling stresses for P's with one edge free and one edge simply supported (F's)

$$\sigma_{cr} = \frac{.416 E}{1 - \nu^2} \left(\frac{t}{b} \right)^2$$

$$\nu = .27 \quad E = 29.6 \times 10^6 \text{ psi}$$

$$\sigma_{cr} = 13.282 \times 10^6 \left(\frac{t}{b} \right)^2$$

Buckling stresses for P's with both edges simply supported (Webs)

$$\sigma_{cr} = \frac{3.29 E}{1 - \nu^2} \left(\frac{t_1}{b_1} \right)^2 = 105.04 \times 10^6 \left(\frac{t_1}{b_1} \right)^2$$

COMBINED BUCKLING LOADS

Once the overall column buckling critical load and the local column buckling loads have been calculated, they should be summarized. The overall column buckling load should be divided by a factor of safety in order to arrive at an allowable buckling load. The calculations for the local buckling loads are already at the allowable values. Then the two loads for the same columns should be compared in the lesser of the two it will be controlling.

In Pier H, of the piles that were examined closely and subsequently analyzed, only one pile (9A) capacity was controlled by local buckling. This pile, however, has a capacity greater than the maximum pile loading of 51.8 kips.

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PIER H

II Overall Column Buckling & III Local Buckling

<u>File</u>	<u>Per</u>	<u>FS</u>	<u>Column P</u>	<u>Local Buckling P</u>
9K	337 ^K	2.14	157 ^K	78.5 ⁺
19D	303		142	71
45K	405		189	94.5
1K	378		177	88.5
9A	365		170	
59A	392		183	91.5
49B	379		177	88.5
34 #1	354		165	82.5
24A	351		164	82
19B	321		150	75

The capacity of the above piles is controlled
in only one instance by local buckling
stress

PIER G

The calculations for Pier G which follow have been carried out in the same manner as those for Pier H. The values for flange and web thicknesses are taken from the URS/Madigan-Praeger report of 1979. No mention was made in this report of the actual bottom depths. Assuming that the bottom depths at this pier are similar to those found at Pier H, the allowable buckling loads shown are too low for the presumed shallower bottom.

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Pier G

Condition II - Column Buckling

$$l = 3 - (12) = 39 \text{ ft} \quad E = E_t \quad l_c = kl \quad k = .506$$

Pile	r	l	$(\pi/l_c)^2$	A_{median}	S_{cr}	P_{cr}
34J	2.59 in	39 ft	.00118	11.303 in ²	27.2 ksi	307 k
34K	2.56		.00115	10.552	27.0	285
48A	2.76		.00134	10.12	28.0	283
49A	2.49		.00109	10.27	26.7	274
60F	2.66		.00124	10.244	27.5	282
61F	2.66		.00124	10.078	27.5	277
69F	2.74		.00132	10.532	27.9	294
69G	2.79		.00137	10.891	28.1	306
20B	2.76		.00134	10.815	28.0	303
20C	2.74		.00132	9.766	27.9	272
25V	2.61		.00120	9.726	27.4	266
26J	2.47		.00107	10.188	26.6	271

Condition III - Local Buckling

Pile	b	t	b/t	b/t	t	b/t	S_{cr}	A	P_{cr}
34J	6 in	.270	22.2 *	10.9	.315	34.6	26.95 ksi	10.482 in ²	282 k
34K		.240	25 *		.312	34.9	21.25	10.124	215
48A		.255	23.5 *		.290	37.6	24.05	9.909	219
49A		.245	24.5 *		.310	35.2	22.13	9.871	218
60F		.225	26.7 *		.312	34.9	18.63	9.366	174
61F		.225	26.7 *		.320	34.1	18.63	9.784	182
69F		.245	24.5 *		.292	37.3	22.13	10.523	233
69G		.218	27.5 *		.352	31.0	17.56	10.455	184
20B		.258	23.3 *		.342	31.8	24.46	10.707	262
20C		.242	24.8 *		.298	36.6	21.59	9.629	208
25V		.232	25.9 *		.305	35.7	19.80	9.651	191
26J		.220	27.3 *		.375	29.1	17.82	10.188	182

* exceeds allowable 16.5

$$S_{cr} = \frac{.416 E}{1 - \nu^2} \left(\frac{t}{b} \right)^2$$

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PIER G

II Overall Column Buckling & III Local Buckling

<u>Pile</u>	<u>P_{cr}</u>	<u>FS</u>	<u>Column P</u>		<u>Local Buckling P</u>
34 J	307 ^K	2.14	143 ^K	71.5 ⁺	282 ^K
34 K	285		133	66.5	215
48 A	283		132	66	219
49 A	274		128	64	218
60 F	282		132	66	174
61 F	277		129	64.5	182
69 F	294		137	68.5	233
69 G	306		143	71.5	184
20 B	303		142	71	262
20 C	272		127	63.5	208
25 V	266		124	62	191
26 J	271		127	63.5	182

Controlling stress in all cases above is critical stress due to overall (Column P) buckling stress.

Conclusions

1. Based on the A.I.S.C. Specifications for the Design, Fabrication and Erection of Structural Steel for Buildings a factor of safety for the overall inelastic column buckling of the pile of 2.14 was used.
2. The factor of safety for local buckling is generally considered as 1.67. This value is incorporated in the expressions used to calculate allowable compressive stress.
3. Using readily available tables, charts and formulae contained in this report, the controlling compressive stress in the subject concrete jacketed H-beams of Piers G and H were calculated.
4. None of the piles sampled in Pier H have capacities below the required 51.8 tons. The pile capacities in this pier were calculated using the actual bottom depths as measured during the 1981 inspection. None of the piles sampled in Pier G appear to be below the desired capacity of 51.8 tons (presuming that Piers G and H have the same imposed loads).
5. With only one examination of the corroded H-piles it is not possible to predict the corrosion rate.

Appendix A

Finite Element Analysis of Column Buckling

ELASTIC BUCKLING ANALYSIS

For: Childs Engineering Corp.
Main Street
Medfield, Mass. 02052

By: Lester M. Cohen
One Essex Road
Medfield, Mass. 02052

Dec. 8, 1981

1.0 Introduction

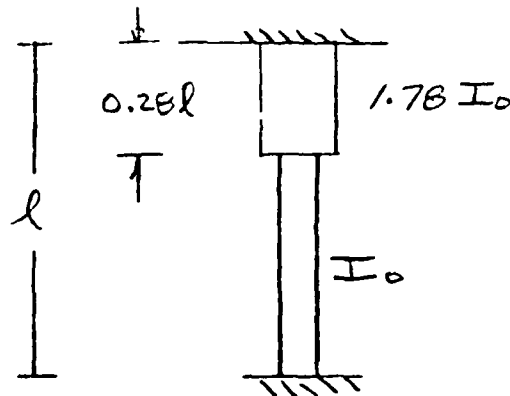
An analytical study is being performed to determine the theoretical buckling capacity of H piles that have been subjected to salt water corrosion. A general purpose finite element program is used. The pile properties (cross sectional area, bending stiffness etc) are determined from data supplied in the URS/Madigan-Proeyer report of Dec. 1, 1978. Pile 60F was chosen as representative. Those data are then used to describe the finite elements of the system. The output of the structural buckling analysis is the critical elastic buckling load of the column in the stressed condition.

The P_{cr} of the column in question is found to be $\sim \underline{361K (180T)}$.

2.0 Discussion

Before actually performing the buckling analysis of the column shown in Fig. 1, a column of uniform I , fixed @ the ends was analyzed. This column (HP 12x53) had $P_{cr, theo} = 5.137 \times 10^5$ lb. P_{cr} as obtained from the finite element analysis which utilized 48 elements along the 532" length was 5.154×10^5 lb for an error of 0.3%.

A second test case was run where the column had the following geometry,



From Roark, 5th edition, page 536 case 1c, after interpolating,

$$P_{cr} \sim 4.6 \frac{\pi^2 E I_0}{l^2} = 3.32 \times 10^5 \text{ lb.}$$

P_{cr} from the finite element analysis yielded 3.38×10^5 lb for negligible error.

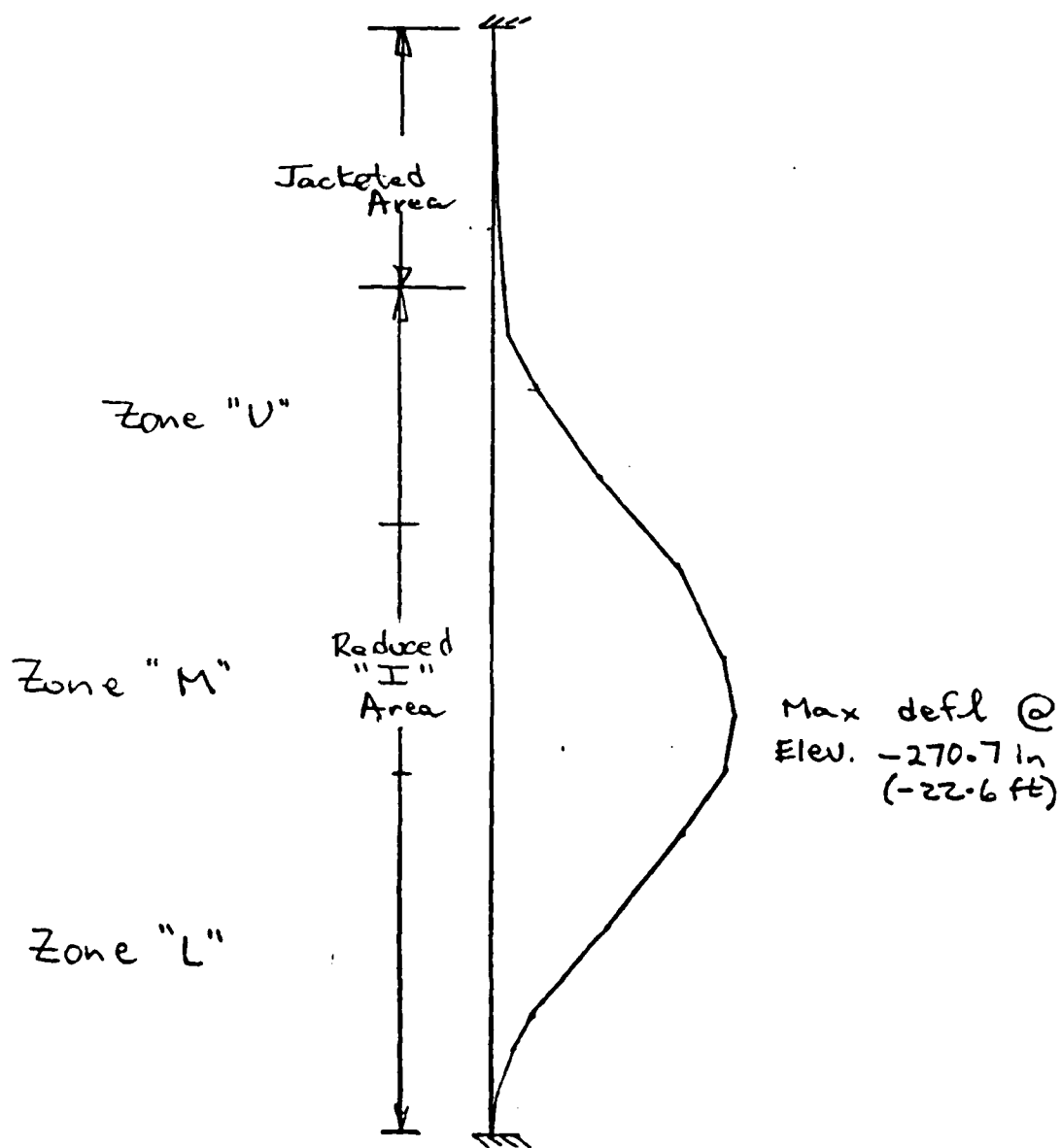
The above shows the accuracy of the finite

element software in solving elastic buckling type problems.

The finite element idealization of column 60F is shown in figure 2. Figures 3 through 6 show the HP cross section @ various areas and the calculations for the reduced area and bending inertias. Table 1 shows the required properties for the finite elements so that the proper stiffnesses can be calculated.

The base of the H section is assumed to be rigidly fixed @ a point 5' below the proposed dredge elev of -37'. The top of the column is assumed to be rigidly fixed at a point 2' above the top of the concrete jacket. This elev. coincides with the center line of a very rigid, 4' deep concrete beam. Therefore the beam is assumed to be the location of column fixity.

The theoretical critical buckling load P_{cr} for the column shown in figure 1 is approx 360,700 lb or 180 tons. This result is from the finite element analysis. The eigenvector (deformed shape) is shown below.



The avg I_{min} of the reduced section is approx 71 in^4 . The length of the reduced section (incl that below the mud line) is $\sim 468''$.

Therefore, the equiv. K for a column of reduced section w/ length 468" is

$$P_{cr} = 360,700 \text{ lb} = \frac{K_{eff} \pi^2 EI}{l^2}$$

$K_{eq} \sim 3.9$

Therefore, if the jacket length is approx. 25% of the overall length and $I_{\text{jacket} + \text{HP}} \gg I_{\text{red HP}}$,

we can approx. Per by

$$P_{cr} \sim \frac{3.9 \pi^2 E I_{avg, red.}}{l_{red.}^2}$$

The I of the reduced section should not change drastically from U to M to L to be approx. valid. The above is reasonable since the jacket almost completely restrains the reduced section for which $K \equiv 4.0$. The small amount of flexibility reduces the K_{eq} below 4.0.

3.0 Conclusions

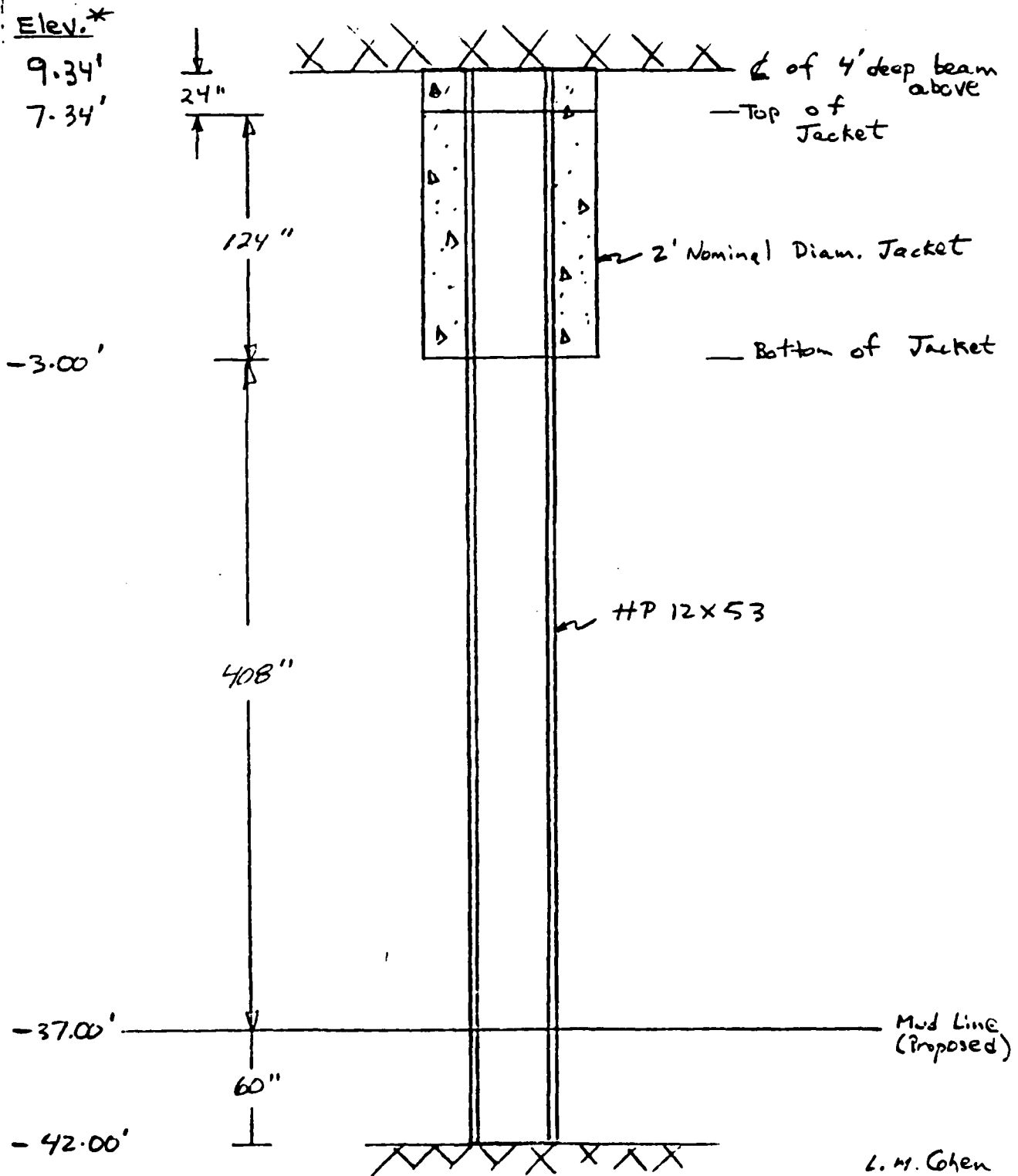
A finite element analysis of column 60F was performed to determine the critical elastic buckling load of the column. P_{cr} was found to be $\sim 361K (180T)$.

Other test cases were performed to validate the type and complexity of the computer model used. Results were excellent.

Due to the severely reduced flange and web thicknesses, a local buckling analysis should be performed as an extension of the present work.

Basic Pile Geometry

Figure 1

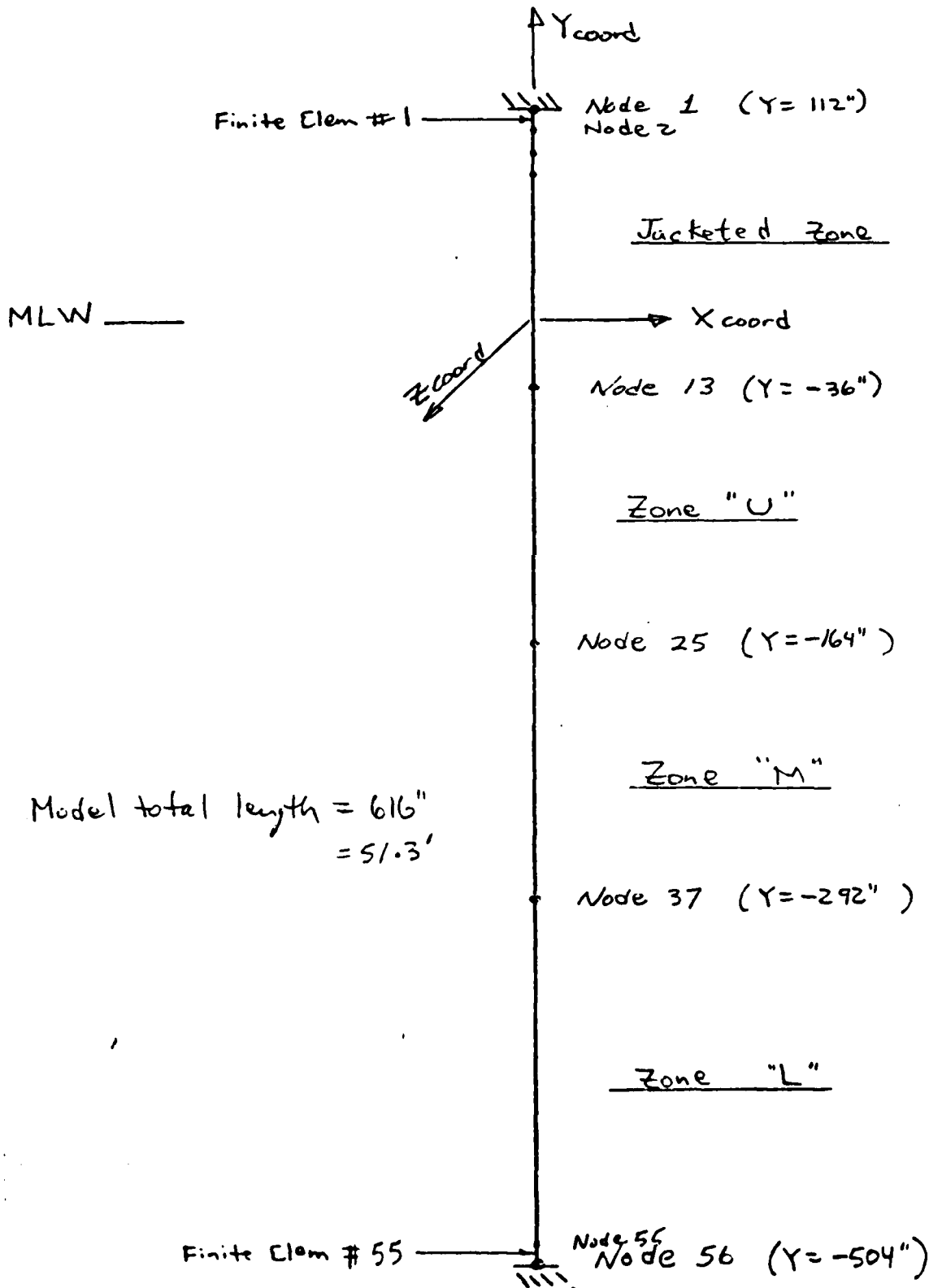


* w/ ref. to Mean Low Water

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Finite Element Idealization

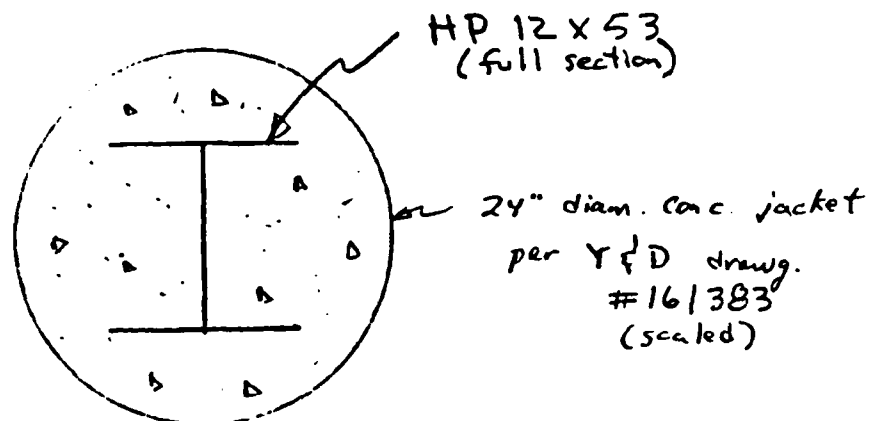
Figure 2



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HP w/ Concrete Jacket Properties

Figure 3



$$I_{conc} \sim \frac{\pi d^4}{64} = 16286 \text{ in}^4$$

modular ratio steel to conc $\rightarrow 10$

$$I_{\text{eff conc jacket}} = 16286/10 \sim 1629 \text{ in}^4$$

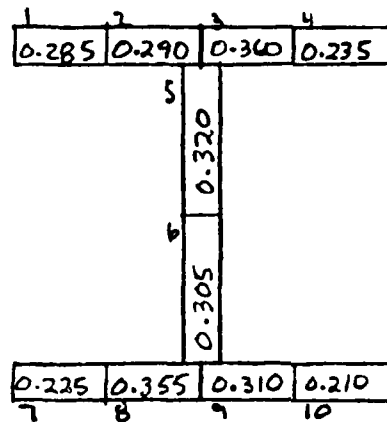
$$A_{conc} \sim \frac{\pi d^2}{4} = 452 \text{ in}^2$$

$$A_{\text{eff. conc jacket}} = \frac{452}{10} \sim 45 \text{ in}^2$$

$$\begin{cases} I_{\text{major}} = I_{HP \text{ orig}} + I_{\text{eff conc}} = 394 + 1629 = 2023 \text{ in}^4 \\ I_{\text{minor}} = \dots = 127 + 1629 = 1756 \text{ in}^4 \end{cases}$$

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Pier 60F
Zone U



shown are
flange and web
thicknesses from
M&P report.

Figure 4

<u>HP Zone</u>	<u>Area</u>	<u>\bar{X}</u>	<u>\bar{Y}</u>	<u>$A\bar{X}^2$</u>	<u>$A\bar{Y}^2$</u>
1	0.86	5.75	4.5		
2	0.87	"	1.5		
3	1.08	"	1.5		
4	0.71	"	4.5		
5	1.81	2.8	0		
6	1.72	"	0		
7	0.68	5.75	4.5		
8	1.07	"	1.5		
9	0.93	"	1.5		
10	0.63	"	4.5		
	<u>10.4 in²</u>			<u>$\Sigma 226 \text{ in}^4$</u>	<u>67+</u>
				<u>43</u>	<u>5</u>
				<u>269 in⁴</u>	<u>72 in⁴</u>

$I_{\text{major}} \sim 269 \text{ in}^4$

$I_{\text{minor}} \sim 72 \text{ in}^4 \leftarrow$

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Pier 60 F
Zone M

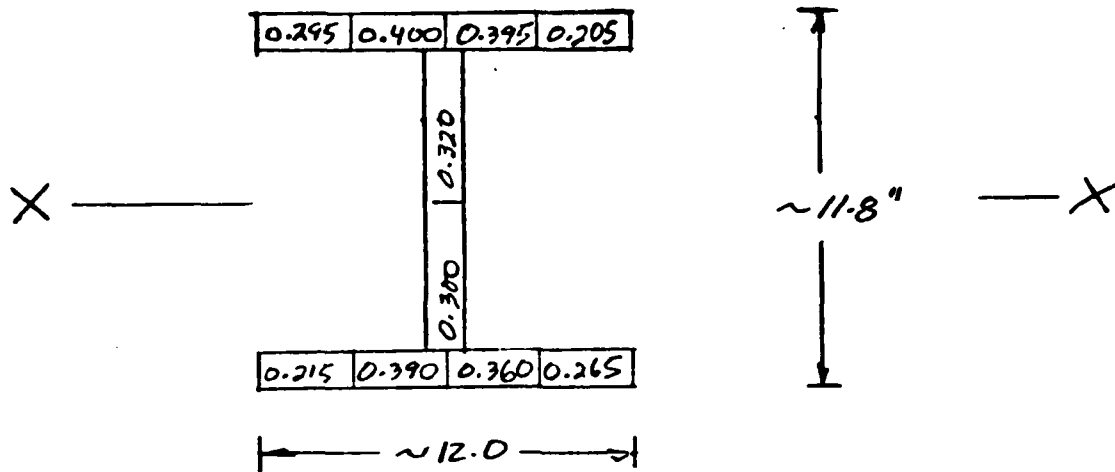


Figure 5

<u>HP Zone</u>	<u>Area</u>	<u>\bar{X}</u>	<u>\bar{Y}</u>	<u>$A\bar{X}^2$</u>	<u>$A\bar{Y}^2$</u>
1	0.89 in ²	5.75	4.5		
2	1.20	5.75	1.5		
3	1.19	5.75	1.5		
4	0.62	5.75	4.5		
5	1.77	2.8	0		
6	1.66	2.8	0		
7	0.65	5.75	4.5		
8	1.17	5.75	1.5		
9	1.08	5.75	1.5		
10	0.80	5.75	4.5		
	<u>11.0 in²</u>			<u>Σ 278</u>	<u>70</u>
		<u>I_o</u>		<u>35</u>	<u>6</u>
				<u>313 in⁴</u>	<u>76 in⁴</u>

$I_{major} \sim 313 \text{ in}^4$

$I_{minor} \sim 76 \text{ in}^4$

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12/7/51

Pier 60F

Zone L

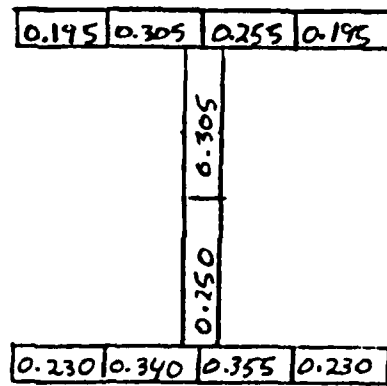


Figure 6

#P	Zone	Area	\bar{X}	\bar{Y}	$A\bar{X}^2$	$A\bar{Y}^2$
1		0.59	5.75	4.5		
2		0.92	"	1.5		
3		0.77	"	1.5		
4		0.59	"	4.5		
5		1.74	2.8	0		
6		1.42	"	0		
7		0.69	5.75	4.5		
8		1.02	"	1.5		
9		1.07	"	1.5		
10		<u>0.69</u>	"	4.5		
		9.5 in ²			Σ 234	60
					I. $\frac{30}{264 \text{ in}^4}$	$\frac{5}{65 \text{ in}^4}$

$I_{\text{major}} \sim 264 \text{ in}^4$

$I_{\text{minor}} \sim 65 \text{ in}^4 \leftarrow$

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Table 1

Required Inputs For Column Properties

$A \equiv$	Cross Sectional Area	in^2
$I_{ZZ} \equiv$	Major Axis Inertia	in^4
$I_{YY} \equiv$	Minor Axis Inertia	in^4
$\text{THICKZ} \equiv$	Depth of Section in Z	in
$\text{THICKY} \equiv$	Depth of Section in Y	in

Refer to Fig 4.4.2 of ANSYS Users Manual. (attached)

\therefore

Defl along minor axis is UZ

Defl along major axis is UX

Defl along col. axis is UY

L M Cohen
12/8/81

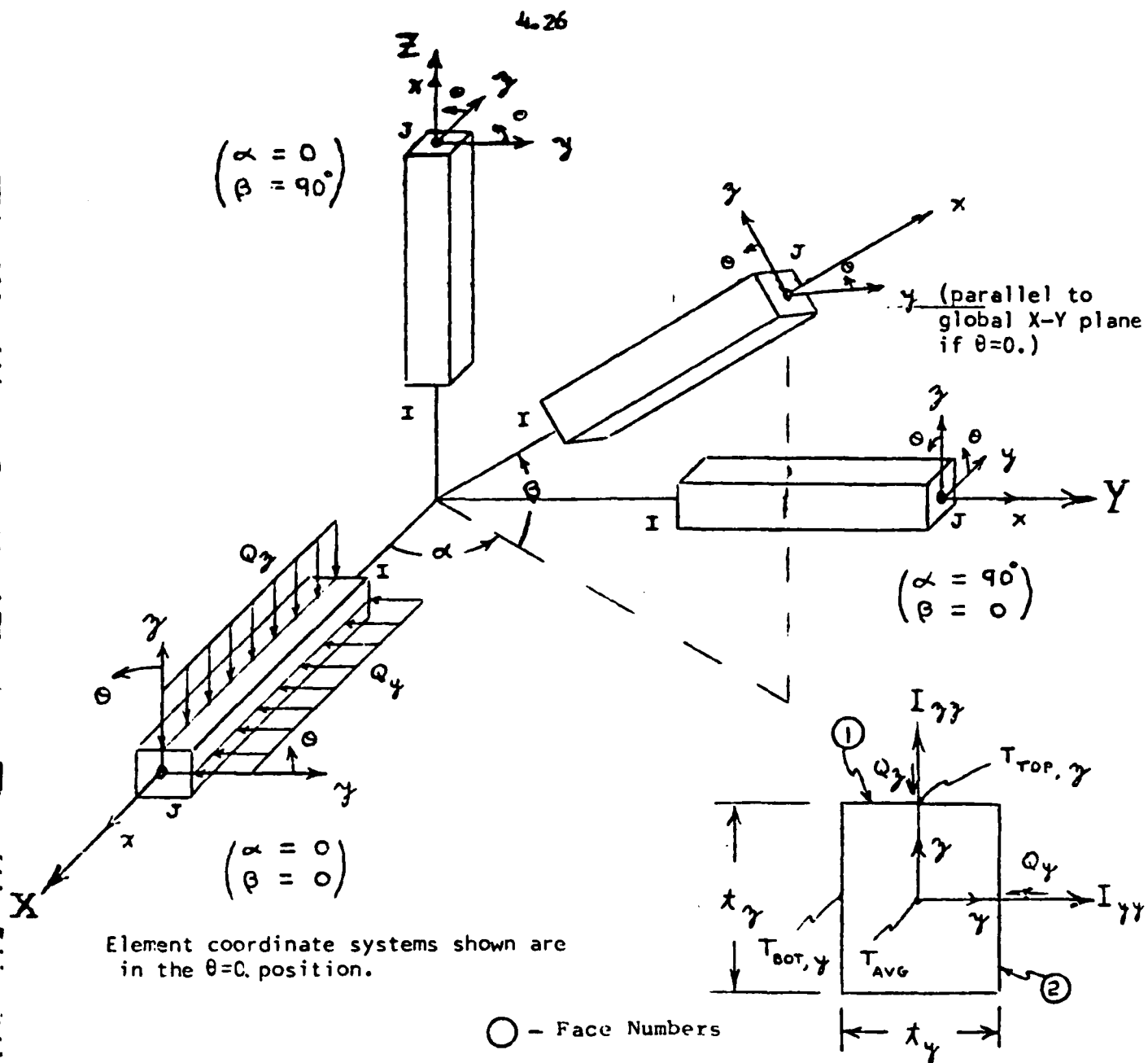


Figure 4.4.1 Three-Dimensional Beam Element

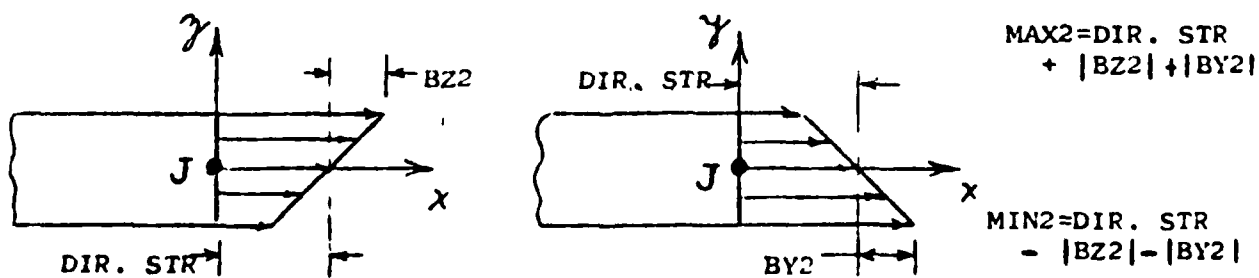


Figure 4.4.2 Three-Dimensional Beam Element Output

Appendix B

Table 34 No. 1e

Formulas for Stress and Strain

Fifth Edition

Raymond J. Roark

Warren C. Young

Published by:

McGraw Hill Book Company, NY

Appendix B

TABLE 34 Formulas for elastic stability of bars, rings, and beams (Cont.)

Reference number, form of bar, and manner of loading and support

1. Stepped straight bar under end load P_1 and intermediate load P_2 ; both ends fixed $P_1 = K_1 \frac{\pi^2 E_1 I_1}{l^2}$ where K_1 is tabulated below



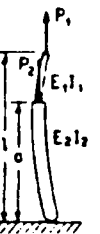
$E_2 I_2 / E_1 I_1$	1.000					1.500					2.000				
	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l
P_2/P_1															
0.0	4.000	4.000	4.000	4.000	4.000	4.389	4.456	4.757	5.354	5.462	4.657	4.836	5.230	6.477	6.834
0.5	3.795	3.246	3.193	3.052	2.749	4.235	3.756	3.873	4.144	3.795	4.545	4.133	4.301	5.204	4.787
1.0	3.572	2.779	2.647	2.443	2.094	4.065	3.211	3.254	3.411	2.900	4.418	3.564	3.648	4.297	3.671
2.0	3.119	2.091	1.971	1.734	1.414	3.679	2.459	2.459	2.452	1.968	4.109	2.766	2.782	3.136	2.496
4.0	2.365	1.388	1.297	1.088	0.857	2.921	1.659	1.649	1.555	1.195	3.411	1.982	1.845	2.008	1.523
8.0	1.524	0.826	0.769	0.623	0.479	1.943	1.000	0.992	0.893	0.671	2.314	1.138	1.141	1.158	0.854

2. Stepped straight bar under tensile end load P_1 and intermediate load P_2 ; upper end free, lower end fixed $P_2 = K_2 \frac{\pi^2 E_1 I_1}{l^2}$ where K_2 is tabulated below



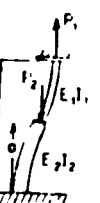
$E_2 I_2 / E_1 I_1$	1.000					1.500					2.000				
	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l
P_2/P_1															
0	9.00	2.25	1.00	0.56	0.36	13.50	3.38	1.50	0.84	0.54	18.00	4.50	2.00	1.25	0.72
0.125	15.55	3.75	1.48	0.74	0.44	21.87	5.36	2.19	1.11	0.65	27.98	6.92	2.89	1.48	0.87
0.250	21.33	5.30	2.19	1.01	0.55	29.51	7.36	3.13	1.53	0.82	37.30	9.31	4.02	2.02	1.10
0.375	29.02	7.25	3.13	1.52	0.74	39.89	9.97	4.37	2.21	1.10	50.10	12.52	5.52	2.86	1.46
0.500	40.50	10.12	4.46	2.31	1.04	55.66	13.92	6.16	3.28	1.60	69.73	17.41	7.71	4.18	2.12

3. Stepped straight bar under tensile end load P_1 and intermediate load P_2 ; both ends pinned $P_2 = K_3 \frac{\pi^2 E_1 I_1}{l^2}$ where K_3 is tabulated below



$E_2 I_2 / E_1 I_1$	1.000					1.500					2.000				
	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l
P_2/P_1															
0	2.60	1.94	1.89	1.73	1.36	2.77	2.24	2.47	2.54	2.04	2.86	2.41	2.89	3.50	2.72
0.125	3.51	2.49	2.43	2.14	1.62	3.81	2.93	3.26	3.18	2.43	3.98	3.21	3.89	4.19	3.24
0.250	5.05	3.41	3.32	2.77	1.99	5.61	4.15	4.64	4.15	2.99	5.99	4.65	5.75	5.52	3.98
0.375	7.71	5.16	4.96	3.76	2.55	8.94	6.61	7.26	5.64	3.82	9.80	7.67	9.45	7.50	5.09
0.500	12.87	9.13	8.00	5.56	3.48	15.72	12.55	12.00	7.90	5.18	17.71	15.45	16.00	10.54	6.87

4. Stepped straight bar under tensile end load P_1 and intermediate load P_2 ; upper end guided, lower end fixed $P_2 = K_4 \frac{\pi^2 E_1 I_1}{l^2}$ where K_4 is tabulated below



$E_2 I_2 / E_1 I_1$	1.000					1.500					2.000				
	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l	a/l
P_2/P_1															
0	10.40	3.08	1.67	1.19	1.01	14.92	4.24	2.21	1.55	1.17	19.41	5.17	2.73	1.88	1.65
0.125	15.57	4.03	2.03	1.40	1.18	21.87	5.57	2.71	1.82	1.57	27.98	7.07	3.16	2.21	1.90
0.250	21.33	5.37	2.54	1.67	1.38	29.52	7.40	3.42	2.20	1.81	37.30	9.34	4.26	2.68	2.24
0.375	29.02	7.26	3.51	2.08	1.67	39.89	9.97	4.50	2.76	2.24	50.10	12.51	5.61	3.39	2.73
0.500	40.51	10.12	4.53	2.72	2.10	55.69	13.91	6.21	3.66	2.84	69.76	17.41	7.76	4.52	3.47

Appendix C

Method for direct calculation
of Critical Buckling Stress in
Engesser portion of
Critical Buckling Curve

Appendix C

"Safe loads for inelastic buckling" - Machine Design, October 3, 1974

Avi Mordkowitz
Project Engineer
Hydraulic Research and Mfg. Co.
Valencia, California

STRESS ANALYSIS is fairly straightforward if a material is not loaded beyond the yield point. But the analysis is complicated in the inelastic range where material behavior is no longer linear.

The analysis of column buckling is particularly cumbersome in the inelastic range. The conventional method is a tedious trial-and-error process in which you first select a slenderness ratio L_e/ρ for the column, assume a value for tangent modulus E_t , and then solve for critical stress σ_{cr} . Then you check a plot of E_t vs. σ to see how accurate you were in your estimate of E_t . Chances are that you weren't close enough, so you have to repeat the process with new values of E_t until your estimated and computed values coincide.

Here is a new approach that eliminates all iteration. It is based on a computed value found from a modified relation of the Euler formula. This value is plotted as a slope on a graph of stress vs. tangent modulus. Critical stress is then read directly from the intersection of this slope with the tangent modulus-curve.

The basic Euler column formula is

$$\sigma_E = \frac{\pi^2 E}{(L_e/\rho)^2}$$

where σ_E = Euler buckling stress, psi; E = modulus of elasticity, psi
 L_e = effective length of column ρ = radius of gyration = $(I_{min}/A)^{1/2}$, in.; I_{min} = minimum moment of inertia of the cross-section in⁴; A = area of cross-section, in²

The Engesser formula introducing the Tangent-Modulus Theory and accounting for inelastic column buckling, is

$$\sigma_{cr} = \frac{\pi^2 E_t}{(L_e/\rho)^2} \quad (2)$$

where σ_{cr} = critical buckling stress, psi.

In this modification, E in Euler equation is simply replaced by E_t the tangent modulus. This modulus is the rate of change of stress σ with respect to strain ϵ , or

$$E_t = \frac{d\sigma}{d\epsilon} \quad (3)$$

where $d\sigma/d\epsilon$ is the slope of the stress-strain diagram from tests, the relationship can be plotted to a large scale, and the slope—or tangent modulus—can be found by locating a straight edge tangent to the curve at the point in question.

The solution of Equation 2 for a column of given material and dimensions involves a trial-and-error process because a value of E_t cannot be selected unless σ_{cr} is known. This limitation can be overcome by the fact that the term $\pi^2 / (L_e/\rho)^2$ in Equation 2 is equivalent to critical strain ϵ_{cr} .

$$\epsilon_{cr} = \frac{\pi^2}{(L_e/\rho)^2} \quad (4)$$

Therefore Equation 2 can be written as

$$\sigma_{cr} = \epsilon_{cr} E_t \quad (5)$$

If you view this equation as being of the classical linear form $y = mx$, where m is the slope of y plotted against x , then the term ϵ_{cr} in Equation 5 is equivalent to the slope of critical inelastic stress σ_{cr} .

Now consider a plot of stress σ vs. E_t . If the value of ϵ_{cr} is plotted as a slope through the origin on this curve, this slope intersects E_t at critical stress σ_{cr} . Thus, σ_{cr} can be read directly, and there is no need for iteration.

The required plots of σ vs. E_t can be found from references such as MIL-HDBK-5B.

Before applying this new approach, you may find it helpful to review common types of end constraints for a column, Fig. 2. In the basic column with both ends pinned, Fig. 2a, the moment at each end is zero. The column with one end free and the other end fixed, Fig. 2b, because of symmetry, behaves in exactly the same way as a pinned column twice as long.

The concept of effective length L_e takes into account these differences in behavior. Effective length is defined as that length of a pinned-end column, Fig. 2a, that would have the same critical load as the column in question. Thus, an endconstraint coefficient α is introduced. Where L = true length of the column, then

$$L_e = \alpha L \quad (6)$$

The effective length for Fig. 2b, therefore, is $L_e = 2L$. In the inelastic range, the critical stress is $\sigma_1/4$ for this case, where σ_1 = critical inelastic stress for the basic, pinned-end column.

In Fig. 2c, the column is fixed at both ends. There are inflection points at the quarter points, causing the center half of the column to behave as a pinned-end column of half the actual length. Therefore, $L_e = L/2$ and $\sigma_{cr} = 4\sigma_1$.

In Fig. 2d, the column is pinned (but constrained to move axially) at one end and fixed at the other. The exact solution gives a critical length value very close to $L_e = 0.7L$, and critical stress is approximately $\sigma_{cr} = 2\sigma_1$.

Example: Consider a column of 7075-T62 aluminum plate, Fig. 3. What is the critical inelastical buckling stress?

Solution: The Euler column formula, Equation 1, is valid if the material response is elastic; that is, stress remains below the proportional limit σ_{PL} . In this example, the Euler equation provides a critical stress $\sigma_E = 131,250$ psi, which is far higher than the proportional limit stress $\sigma_{PL} = 61,000$ psi. So an inelastic analysis must be applied.

The tangent-modulus vs. stress curve is obtained from a reference, Fig. 4. The column loading is seen to be equivalent to that of Fig. 2d with end-constraint coefficient $\alpha = 0.7$. From Equation 6 the effective length $L_e = L\alpha = (0.7) (1 \text{ ft}) (12 \text{ in./ft}) = 8.4 \text{ in.}$ The cross-sectional area $A = 0.6 \text{ in.}^2$ The minimum moment of inertia $I_{min} = bh^3/12 = 0.054 \text{ in.}^4$ Radius of gyration $\rho = (I_{min}/A)^{1/2} = 0.3 \text{ in.}$

$$\begin{aligned} \epsilon &= \pi^2 / (L_e / \rho)^2 = \pi^2 / (8.4 / 0.3)^2 \\ &= 0.0125 \text{ in./in.} \end{aligned}$$

Thus, Equation 5 has the form $\sigma_{cr} = 0.0125 E_t$. If this equation is plotted on Fig. 4 (in other words, if a straight line of equivalent slope is drawn through the origin), the resulting line intersects the E_t curve at the critical stress σ_{cr} . In this case the value is $\sigma_{cr} = 72,000$ psi, and the problem is thus solved without iteration.

This critical stress happens to be below the compressive yield strength σ_{cy} of 80,000 psi as indicated by Fig. 4. However, the "yield strength" is a value determined by the arbitrary standard of a 0.2% offset, and this offset may incorporate inelastic behavior. Therefore, computed inelastic loads can fall in a range where behavior is ordinarily assumed to be elastic.

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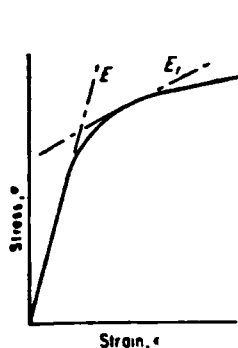


Fig. 1—Representation of tangent modulus.

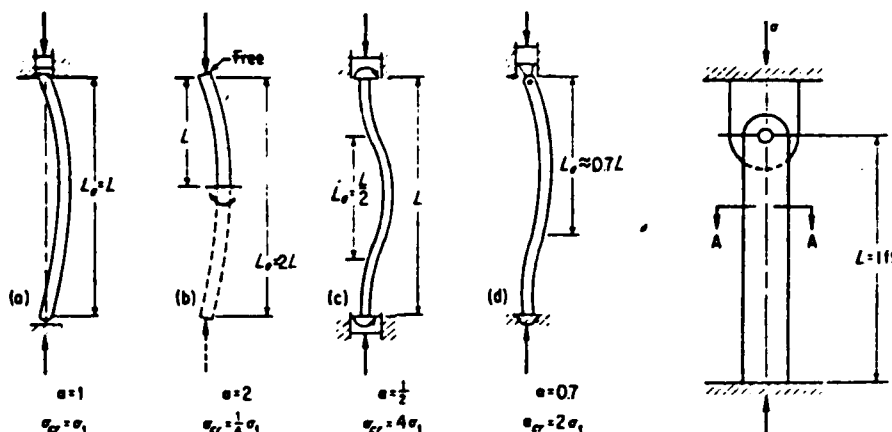


Fig. 2—Configurations for columns.

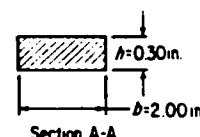


Fig. 3—Column configuration for example problem

the rate of change of stress σ with respect to strain ϵ , or

$$E_1 = \frac{d\sigma}{d\Omega} \quad (3)$$

where $d\sigma/d\epsilon$ is the slope of the stress-strain diagram at a particular point, Fig. 1.

the stress-strain relationship obtained directly from tests, the relationship can be plotted to a true scale, and the slope—or tangent modulus—can be found by locating a straight edge tangent to the curve at the point in question.

... solution of Equation 2 for a
... of given material and di-
... involves a trial-and-error
... because a value of E_r can-
... selected unless σ_{cr} is known.
... can be overcome by
... that the term $\pi^2/(L_r/\rho)^2$
... Equation 2 is equivalent to criti-
... strain σ_{cr} .

$$\frac{\pi^2}{(L_c/\rho)^2} \quad (4)$$

Before Equation 2 can be

E. (5)

view this equation as being in the classical linear form $y = mx + c$ where m is the slope of y against x , then the term

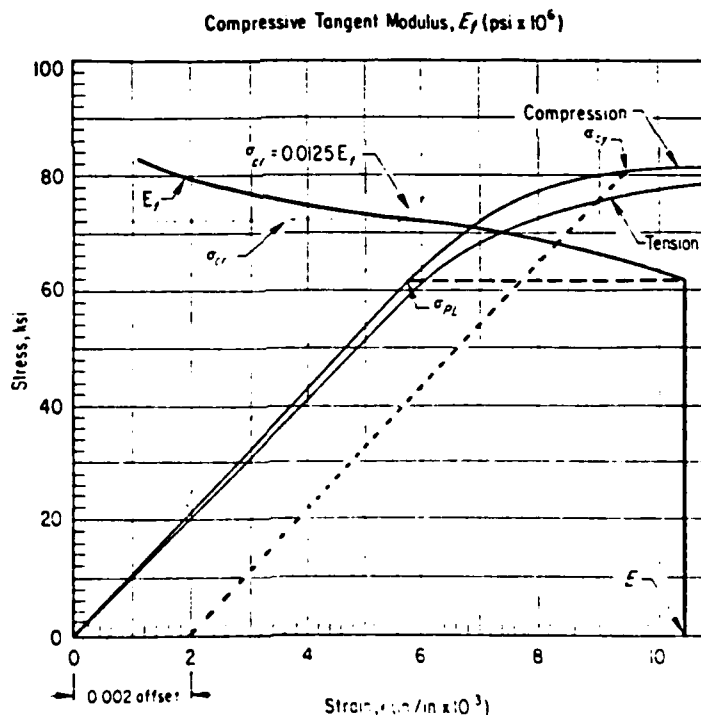


Fig. 4—Property data used in graphical solution of example problem.

r_{cr} in Equation 5 is equivalent to the slope of critical inelastic stress σ_{cr} .

Now consider a plot of stress σ vs. E_p . If the value of r_c is plotted as a slope through the origin on this curve, this slope intersects E_p at critical stress σ_c . Thus, σ_c can be read directly, and there is

no need for iteration.

The required plots of σ vs. E_r can be found from references such as MIL-HDBK-5B.

Before applying this new approach, you may find it helpful to review common types of end constraints for a column, Fig. 2. In the basic column with both ends

Appendix D

Buckling Stress values
and corresponding
Tangent Modulus values

CHILDS ENGINEERING CORPORATION

Box 333
MEDFIELD, MA 02052

JOB _____

SHEET NO _____

OF _____

CALCULATED BY KMC

DATE 27 Dec 81

CHECKED BY _____

DATE _____

SCALE _____

σ_c	E_z
25.00×10^3	29.6×10^6
25.23	28.98
25.46	28.38
25.68	27.80
25.89	27.22
26.10	26.66
26.29	26.09
26.48	25.55
26.66	25.02
26.83	24.48
27.00	23.98
27.31	22.97
27.60	22.02
27.87	21.11
28.13	20.26
28.37	19.45
28.59	18.67
28.79	17.92
28.98	17.22
29.16	16.55
29.33	15.91
30.03	13.18
30.55	11.05
30.95	9.37
31.26	8.03
31.70	6.07
32.00	4.79
32.23	3.57
32.35	2.72
32.55	2.18
32.66	1.81
32.78	1.54
32.87	0.98

σ_c	E_z
32.84×10^3	0.80×10^6
32.89	0.51
32.93	0.36
32.96	0.20
32.98	0.13
33.00	0.13

$$\sigma_y = 33 \times 10^3 \text{ psi}$$

$$\sigma_f = 25 \times 10^3 \text{ psi}$$

These limits approximate
A-7 steel

Appendix E

Required capacity of piles
in Piers G and H

See Childs Engineering Corporation
report of condition of piers at
Charleston Naval Shipyard for details

Childs Engineering Corporation finds maximum LL+DL on Piers H and G, piles are as listed below

@Charleston Naval Station.

A+K	piles	43.6 tons
B	piles	40.9 tons
D	piles	51.8 tons
C	piles	tons
F	piles	tons
G	piles	tons
J	piles	tons
V	piles	tons
#1	piles	40.9 tons

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Addendum

General Review Comments on the "Analysis of the Remaining Strength of Concrete Jacketed Steel H-Piles" Report

1. Factors of Safety. The narrative is not clear on how the value of .506 for the factor of safety was arrived at.

2. Condition I. The equation for sigma critical (σ_{cr}) is off somewhat, it should be,

$$\sigma_{cr} = \frac{\pi^2 E}{(K l/r)^2}$$

There exists some confusion among the "K" in this equation and the various "Ks" on the "overall column buckling" worksheet. Which "K" value is this?

3. Condition III. The coefficient for sigma critical (σ_{cr}) for flanges appears to be incorrect. It should be 0.35 instead of 0.416. Refer to Table 26 in "Buckling Strength of Metal Structures" by Bleich, this table is enclosed for your convenience within enclosure (3).

4. Finite Element Analysis of Column Buckling (see Fig. 3). The derivation for the moment of inertia for a composite concrete-steel pile assumes that the concrete acts in tension, which it cannot. Additionally, the assumption of adequate bond, and, therefore, shear transfer between the steel and concrete may not be valid. Recalculating the composite moment of inertia considering that concrete acts in compression only results in a lower value. This will affect the stiffness used in the finite element analysis and may impact the conclusion that the unbraced column length is reduced to that length below the concrete jacket. These points should be considered to determine if the column load carrying capacity remains acceptable.

5. Paginate the report.

1. Factors of Safety

The factor .506 is not a factor of safety but rather a coefficient related to the end restraints of the column in question. As used in the basic Euler formula ($P_{cr} = K \frac{\pi^2 EI}{l^2}$), the factor K for an idealized column with both ends fixed against rotation is 4.0. If the modern and common relationship for critical stress is used ($\sigma_{cr} = \frac{\pi^2 E}{(l_e/r)^2}$), the effective length, l_e , is determined by using a factor K' times the actual length of the column ($l_e = K'l$). This factor also reflects the end restraints of the column in question. An idealized column fixed against rotation at both ends has an effective length of $\frac{1}{2}l$ or, in other words, $K' = 0.5$. Since the relationships for P_{cr} and σ_{cr} are of the same form, the relationship between K and K' can be shown as follows: $\sigma_{cr} = \frac{\pi^2 E}{(l_e/r)^2} = \frac{\pi^2 E}{(K'l/r)^2} = \frac{\pi^2 E}{(K')^2 (l/r)^2} = K \frac{\pi^2 E}{(l/r)^2}$. Therefore, $K = \frac{1}{(K')^2}$.

In the original discussion, a value for K of 3.9 was used to reflect the small amount of rotation in the column which the concrete jacket would allow.

$$\text{If } K = 3.9, K' = \frac{1}{\sqrt{K}} = \frac{1}{\sqrt{3.9}} = .506.$$

2. The equation for σ_{cr} should be written $\sigma_{cr} = \frac{\pi^2 E}{(K'l/r)^2}$ or $K \frac{\pi^2 E}{(l/r)^2}$. See the discussion in 1. above.

3. In his discussion of Elastic Stability of Plates and Shells, Roark¹ sites two conditions applicable to the flanges of H-beams. One condition considers the loaded edges simply supported, one unloaded edge free and the other clamped. The other condition considers the loaded edges simply supported, one unloaded edge free and the other simply supported. We have used the conservative relationship of assuming one unloaded edge simply supported and the other edge free. Actually, the restraint of the edge of the flange supported by the web of the H-beam lies between simple and rigid support. The value of the constant +0.416 in the relationship $\sigma_{cr} = \frac{0.416E}{1-\nu^2} \left(\frac{t}{b}\right)^2$ assumes that the attached edge is simply supported and that the ratio of length of flange being considered is five times greater than the flange width. (See Table XVI attached). Of all the values shown, this was the most conservative.

Actually, the section of the pile where the greatest flange area reduction occurs is most often less than two feet long ($a/b = 4$ for a 12 inch H-pile). In calculating critical buckling stress in the flange of an H-beam column. For practical considerations, local buckling seldom contributes to final failure of a steel H-section marine pile. Brackett et al.² found in their studies that as the local area of corrosion of a pile shortened, it had less influence on the overall strength of the pile. It is this writer's opinion that using a factor of 0.35 instead of 0.416 would indicate undue influence of local buckling on overall column strength.

4. The two citations given concerning concrete jacketing of steel members both are concerned with beams which are subject to bending. In composite beams, the horizontal shear must be carried from steel to concrete. At the interface, some sort of shear transfer member must³ be used if any significant amount of shear is to be transferred. Where beams are encased in concrete they are still carrying bending. The requirement in Merritt⁴ was for mesh reinforcing within the concrete. This mesh would not transfer horizontal shear, but would keep the concrete from cracking due to thermal stresses. Columns are primarily compression carrying members. Bending stress is the result of buckling prior to failure and is not a principal reaction to loading.

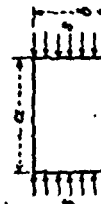
The overall question of the action of a column in compression should be addressed. The action of the column is primarily one of compression. Critical loading is considered to be that axial load which will keep a column in a bent position after the column has been deflected by a hypothetical external lateral force after the lateral force is removed. The entire cross section of the column is still in compression, the magnitude of compressive load varies and thus generates bending moment.

There is no tension in the column when it is at or below this critical buckling load. The calculations on the following pages address load transfer through shear from the concrete jacket to the steel H-pile and vice versa. These calculations also show why we believe there is no tensile stress generated in the concrete jacket.

1. Roark, R.J. - Formulas for Stress and Strain
4th Edition, McGraw-Hill Book Company,
New York, NY 1965
2. Brackett, R.L., Nordell, W.J.Ph.D, and Rail, R.D. -
Inspection Requirements Analysis and Nondestructive
Testing Technique Assessment for the Underwater
Inspection of Waterfront Facilities, U.S. Navy
Civil Engineering Laboratory
3. Gaylord, Edwin H., Jr. and Gaylord, Charles N. -
Structural Engineering Handbook, McGraw-Hill Book
Company, New York, NY 1968
4. Merritt, Frederick S. - Building Construction
Handbook, McGraw-Hill Book Company, New York, NY
1958

TABLE XVI.—FORMULAS FOR ELASTIC STABILITY OF PLATES AND SHELLS
 E = modulus of elasticity; ν = Poisson's ratio; a = longer dimension, b = shorter dimension for all rectangular plates, t = thickness for all plates and shells. All dimensions in inches, all forces in pounds, all angles in radians. Compression positive; tension negative.

Form of plate or shell and manner of loading	Manner of support	Formula (for critical unit compressive stress s_c , unit shear stress s_s , load P , bending moment M , or unit external pressure p at which elastic buckling occurs)
A. Rectangular plate under equal uniform compression on two opposite edges b	1. All edges simply supported	$s_c = K \frac{E}{1-\nu^2} \left(\frac{t}{b} \right)^2$ Here K depends on ratio a/b and may be found from the following table: $\frac{a}{b} = 0.2 \quad 0.3 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \quad 1.3 \quad 1.4 \quad 1.6 \quad 1.8 \quad 2.0 \quad 2.3 \quad 2.4 \quad 2.7 \quad 3.0$ $K = 27.2 \quad 10.9 \quad 6.92 \quad 4.23 \quad 3.45 \quad 3.20 \quad 3.40 \quad 3.68 \quad 3.43 \quad 3.33 \quad 3.26 \quad 3.35 \quad 3.40 \quad 3.32 \quad 3.26$ (See unequal end compressions, see Ref. 33) (Ref. 1, 6)
	2. All edges clamped	$s_c = K \frac{E}{1-\nu^2} \left(\frac{t}{b} \right)^2$ $\frac{a}{b} = 1 \quad 2 \quad 3 \quad \infty$ $K = 7.7 \quad 6.7 \quad 6.4 \quad 5.75$ (Ref. 1, 6, 7)
	3. Edges b simply supported, edges a clamped	$s_c = K \frac{E}{1-\nu^2} \left(\frac{t}{b} \right)^2$ $\frac{a}{b} = 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.8 \quad 1.0 \quad 1.3 \quad 1.4 \quad 1.6 \quad 1.8 \quad 2.0$ $K = 7.78 \quad 6.32 \quad 5.80 \quad 5.76 \quad 5.00 \quad 5.32 \quad 5.80 \quad 5.76 \quad 5.00 \quad 5.80 \quad 5.75$ (Ref. 1, 6)
	4. Edges b simply supported, one edge a simply supported, other edge a free	$s_c = K \frac{E}{1-\nu^2} \left(\frac{t}{b} \right)^2$ $\frac{a}{b} = 0.5 \quad 1.0 \quad 1.3 \quad 1.4 \quad 1.6 \quad 1.8 \quad 2.0 \quad 2.5 \quad 3.0 \quad 4.0 \quad 5.0$ $K = 3.63 \quad 1.16 \quad 0.934 \quad 0.784 \quad 0.687 \quad 0.622 \quad 0.574 \quad 0.502 \quad 0.464 \quad 0.435 \quad 0.416$ (Ref. 1)
	5. Edges b simply supported, one edge a clamped, other edge a free	$s_c = K \frac{E}{1-\nu^2} \left(\frac{t}{b} \right)^2$ $\frac{a}{b} = 1 \quad 1.3 \quad 1.4 \quad 1.6 \quad 1.8 \quad 2.0 \quad 2.5 \quad 3.0 \quad 4.0 \quad 5.0 \quad 6.0$ $K = 1.40 \quad 1.29 \quad 1.21 \quad 1.18 \quad 1.12 \quad 1.10 \quad 1.09 \quad 1.09 \quad 1.10 \quad 1.12 \quad 1.14 \quad 1.16 \quad 1.19 \quad 1.21$ (Ref. 1)
	6. Edges b clamped, edges a simply supported	$s_c = K \frac{E}{1-\nu^2} \left(\frac{t}{b} \right)^2$ $\frac{a}{b} = 0.5 \quad 0.8 \quad 1.0 \quad 1.3 \quad 1.4 \quad 1.6 \quad 1.8 \quad 2.0 \quad 2.5 \quad 3.0$ $K = 11.0 \quad 7.16 \quad 6.64 \quad 6.00 \quad 4.49 \quad 4.39 \quad 4.30 \quad 4.30 \quad 4.30 \quad 4.30 \quad 4.30$ (Ref. 1)



B. Rectangular plate under uniform compression on edges a and b and uniform tension on edges a and b .

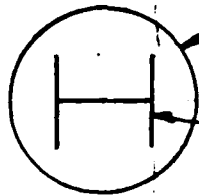
7. All edges simply supported

$$s_c = \frac{m^2}{a^2} + \frac{n^2}{b^2} = 0.823 \frac{E}{1-\nu^2} \left(\frac{t}{b} \right)^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)$$

Here m and n signify the number of half-waves in the buckled plate in the x and y directions, respectively. To find s_c for a given a/b , take $m = 1, n = 1, 2, 3, \dots$ and $s_c < s_c$ where $s_c = \frac{0.823 E}{1-\nu^2} \left(\frac{t}{b} \right)^2$. If s_c is too large to satisfy this inequality, take $m = 1, n = 2, 3, \dots$ and so on.

Bending & Shear

Horizontal Shear



1. 24" ϕ Concrete jacket

2. 12x12 H-pile (HP12x53)

3. No reinforcing in concrete,
no bonding (other than
adhesion) between steel
and concrete.

$$f_v = \frac{VQ}{It}$$

Let $f_v = 120$ psi (allowable bond for
plain reinforcing
bars)

$$I_c = \frac{\pi d^4}{64} = 16286 \text{ in}^4$$

$$I_s = 127.3(10) = 1273 \text{ in}^4$$

$$EI = 17559 \text{ in}^4$$

$$Q = 88.4(8.4) = 742.6 \text{ in}^3$$

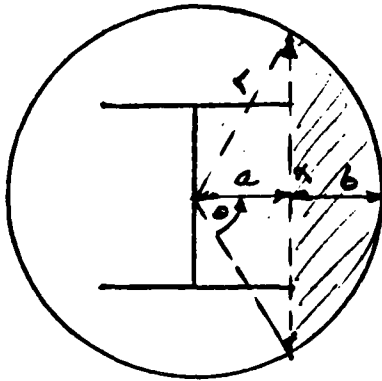
$$V = \frac{f_v It}{Q} = \frac{(120) 17559(12)}{742.6} = 34049 \text{ lbs}$$

34 K @ face of flange of
HP12 allowable
re ACI code

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Area of shaded area

$$a = 6"$$

$$r = 12"$$

$$\cos \theta = \frac{6}{12}$$

$$\theta = 60^\circ$$

$$Area' = \pi r^2 \frac{60^\circ}{360}$$

$$= \pi (12)^2 \frac{1}{3} = 150.8 \text{ in}^2$$

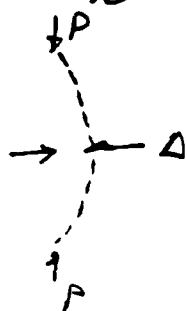
$$Area = Area' - 2 \left[\frac{12^2 - 6^2}{2} (12 - 6) \right] = 88.4 \text{ in}^2$$

$$\text{Centroid (Using parabola relation)} = a + \frac{2}{5} (12 - 6) = 8.4"$$

Shearing force on pile/jacket.

- (I) Assume pile deflection equals allowable bend from mill specifications = $\frac{1}{8}$ " in 10'

$$\Delta = \frac{39}{10} \left(\frac{1}{8} \right) = 0.4875 \text{ in}$$



Use $P = P_{cr}$ $M = P\Delta$

$$l = 39'$$

$$K = 3.9$$

$$I = 127.3$$

$$E = E_t$$

$$r = 2.86$$

$$K' = k = .506$$

$$C_c = 133.1$$

$$\frac{\pi^2}{\left(\frac{kl}{r} \right)^2} = \frac{\pi^2}{(82.8)^2} = .00144$$

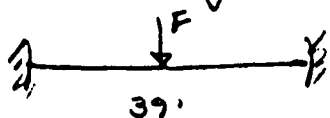
$$\frac{kl}{r} = \frac{(.506)(39)(12)}{2.86} = 82.8 \text{ Use } E_t$$

$$\sigma_{cr} = 28.4 \text{ ksi}$$

$$\sigma_{cr} A = P_{cr} = (15.6)(28.4) = 443 \text{ k}$$

$$M = (443)(.4825) = 216 \text{ k-in}$$

Equate moment to moment generated by force at midlength of beam fixed at both ends



$$M = \frac{Fl}{8} = 216 = \frac{F(39)(12)}{8}$$

$$F = 3.69 \text{ k}$$

$$V = F/2 = 1.84 \text{ k} < 34 \text{ k}$$

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Shearing force on pile/jacket

② Use limit = $\frac{P_{cr}}{A} + \frac{P_{cr} \Delta}{S} > f_y$

$P_{cr} \Delta = M$

$\frac{P_{cr}}{A} = \frac{443}{15.6} + \frac{M}{21.1} = 33$

$M = 97 \text{ k-in} < 216 \text{ k-in in Case I}$

Does not limit

The bonding strength of 120 psi for 3000 psi concrete is not exceeded in either case.

Conclusion: No shear studs are needed to transfer loads from concrete to steel.

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Does tensile stress in concrete exceed allowable stress for plain concrete

$$\frac{P}{A} - \frac{M}{S} = \text{tensile stress}$$

$$P = P_{cr} = 443 \text{ k}$$

Use transformed section

$$A = \pi(12)^2 + 15.6(10) = 608 \text{ in}^2$$

$$\frac{P}{A} = \frac{443}{608} = 0.729 \text{ ksi}$$

$$M = 216 \text{ k-in}$$

$$S = \frac{I}{C} = \frac{17559}{12} = 1463 \text{ in}^3$$

$$\frac{M}{S} = \frac{216}{1463} = 0.148 \text{ ksi}$$

$$0.729 - 0.148 = 0.581 \text{ ksi, but this is compression}$$

{ There is no tension, column load
Prestresses concrete jacket.

END

DTIC

6-86